

* Non Calc portion
or
Calc portion \rightarrow Exact answer
is a distractor

12 Linear Approximation

Same as
 (Δy)

$$f'(a) = \lim_{\Delta x \rightarrow 0} \frac{f(a + \Delta x) - f(a)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}$$

Slope of tan line
to f at a

Note = b/c of limit

$$f'(a) \approx \frac{\Delta f}{\Delta x} \quad * \text{note } \approx \text{ b/c no limit}$$

$$\rightarrow \Delta f \approx f'(a) \cdot \Delta x$$

Linear Approximation (AKA Tan line approx)



Differential Notation

exact tan line approx	$y = f(x)$ $\rightarrow dy = f'(x) \cdot dx$ $\rightarrow \Delta y \approx f'(x) \frac{\Delta x}{\Delta x}$	$\rightarrow \frac{dy}{dx} = f'(x)$
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Tan line  pictures agree at close $(a, f(a))$
at $x=a + \Delta x$

$$y - f(a) = f'(a)(x - a)$$

$\Delta y \approx f'(a) \frac{\Delta x}{\Delta x}$

$$\sqrt[3]{8.9} \rightarrow \frac{9}{8}$$

Example 1: Use a linear approximation to find $\sqrt[3]{8.1}$.

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\text{Want } f(8.1) = \sqrt[3]{8.1}$$

tan line to f at $x=8$

$$f(8) = \sqrt[3]{8} = 2$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

$$(8, 2) \quad m = \frac{1}{12}$$

tan line

$$y \approx \frac{1}{12}(x-8) + 2$$

$$y(8.1) \approx \frac{1}{12}(8.1-8) + 2$$

approx

$$y(8.1) \approx 2.00833$$

exact

$$f(8.1) = 2.00829$$

Ex 2) Use a linear approximation to find $f(3.02)$

A) $f(x) = x^2$

tan line at $x=3$

$$f(3) = 9$$

$$f'(3) = 6$$

$$f'(x) = 2x$$

$$f'(3) = 6$$

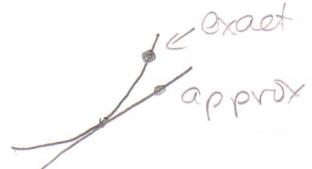
$$y = 6(x-3) + 9$$

$$\text{tan line approx } y(3.02) \approx 6(3.02-3) + 9$$

$$y(3.02) \approx 9.12$$

exact

$$f(3.02) = 9.1204$$



want approx $f(3.02)$

B) $f(x) = x^{-1} = \frac{1}{x}$ tan line egn at $x=3$
 $f'(x) = -x^{-2} = \frac{-1}{x^2}$ $f(3) = \frac{1}{3}$
 $f'(3) = -\frac{1}{9}$

approx $y = -\frac{1}{9}(x-3) + \frac{1}{3}$
actual $y(3.02) \approx 0.331111$
 $f(3.02) = 0.331126$

C) $f(x) = \tan(\frac{\pi}{3} \cdot x)$ tan line egn at $x=3$
 $f(3) = \tan(\pi) = 0$ $f(3) = 0$
 $f'(x) = \sec^2(\frac{\pi}{3} \cdot x) \cdot \frac{\pi}{3}$ $f'(3) = \frac{\pi}{3} (\sec(\pi))^2 = \frac{\pi}{3}$

approx $y = \frac{\pi}{3}(x-3) + 0$ under estimate
exact $y(3.02) \approx 0.020944$ bigger
 $f(3.02) = 0.020947$

$y(2.98) \approx$
 $f(2.98) =$

D) $f(x) = x^4$ tan egn at $x=3$
 $f(3) = 3^4 = 81$ $f(3) = 81$
 $f'(x) = 4x^3$ $f'(3) = 108$
 $f'(3) = 4(3)^3 = 108$

approx $y = 108(x-3) + 81$
exact $y(3.02) \approx 33.16$
 $f(3.02) = 33.18$

E) $f(x) = \frac{1}{x+1} = (x+1)^{-1}$ tan line eqn at $x=3$

$$f(3) = \frac{1}{3+1} = \frac{1}{4}$$

$$f'(x) = -1(x+1)^{-2}, 1$$

$$f'(x) = \frac{-1}{(x+1)^2}$$

$$f'(3) = \frac{-1}{(3+1)^2}$$

$$y = -\frac{1}{16}(x-3) + \frac{1}{4}$$

approx $y(3.02) \approx 0.24875$
 actual $f(3.02) = 0.248756 \leftarrow \text{bigger}$

Error Percentage

you scored $2\frac{1}{2}\%$

Error : $27 - 21 = 6$

Ex E : Error : $\frac{27-21}{27} \times 100 =$

Error % : $\frac{.000006}{.248756} \times 100 = .002412\%$