13. Extreme Values of a function

Absolute (global) max/min (hill, valley) – $f(c)$ is an absolute max if $f(c) \geq f(x)$
min $f(c) \leq f(x)$
for all x in the domain of f.

Relative (local) max/min – $f(c)$ is a relative max if $f(c) \geq f(x)$
min $f(c) \leq f(x)$
for all x in some open interval <u>near</u> c.

Extreme Value Theorem

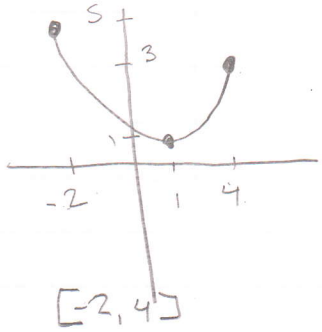
If f is <u>CTS</u> on a <u>closed interval</u> $[a,b]$, then f has both a max and min.

Critical point:

$x=c$ is a critical point of f if $\underbrace{f'(c)=0}_{\text{stationary pt}}$ or <u>$f'(c)$ does not exist</u>

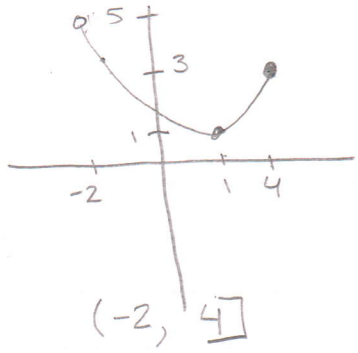
* On a <u>closed</u> interval (or restricted domain) always check the endpoints.

Example 1: Find the max/min



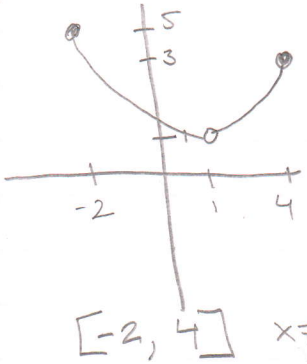
$[-2, 4]$

max: $(-2, 5)$
min: $(1, 1)$



$(-2, 4]$

max: no max
min: $(1, 1)$



$[-2, 4]$ $x \neq 1$

max: $(-2, 5)$
min: no min

Example 2 Find the extreme values for each fn

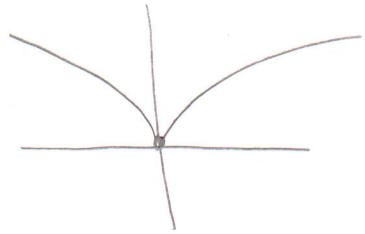
A) $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$ $\longrightarrow f'(x) = 0$ $\quad 2 \neq 0$

$\longrightarrow f'(x)$ DNE $\quad 3\sqrt[3]{x} = 0$

$x = 0$

Sketch f'



min at $x = 0$
min is $y = 0$

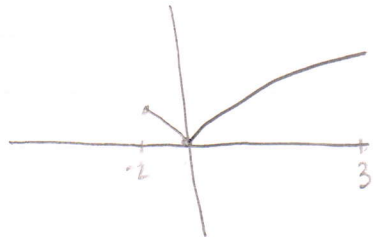
$(0, 0)$

B) $f(x) = x^{2/3}$ $[-2, 3]$

$f(-2) \approx 1.58$
$f(3) \approx 2.08$

min $(0, 0)$
max $(3, 2.08)$

Sketch f2



C) $f(x) = \frac{1}{\sqrt{4 - x^2}}$ Interval $(-2, 2)$ $\quad f(x) = \sqrt{4 - x^2}$ $[-2, 2]$

$f(x) = (4 - x^2)^{-1/2}$

$f'(x) = -\frac{1}{2}(4 - x^2)^{-3/2}(-2x)$

$= \frac{\cancel{-2}x}{\cancel{-2}\sqrt{4 - x^2}^{\,3}}$

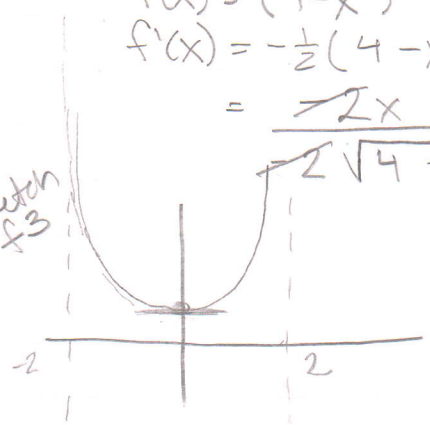
$f'(x) = 0 \rightarrow \boxed{x = 0}$ CP

$f'(x)$ DNE $\rightarrow \sqrt{4 - x^2} = 0$

$4 - x^2 = 0$

$\boxed{x = \pm 2}$

Sketch f3

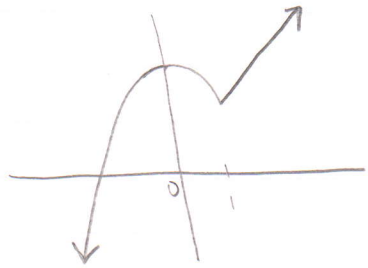


min $(0, \frac{1}{2})$

$f(0) = \frac{1}{\sqrt{4 - 0^2}} = \frac{1}{2}$

D) $f(x) = \begin{cases} 5 - 2x^2 & x \le 1 \\ x + 2 & x > 1 \end{cases}$

f4



no more = unless you show f' is cts

$f'(x) = \begin{cases} -4x & x < 1 \\ 1 & x > 1 \end{cases}$

CP: $f'(x) = 0$ $\quad -4x = 0$ or $1 \neq 0$

$\boxed{x = 0}$

$f'(x)$ DNE (@boundary) $\quad \lim_{x \to 1^-} -4x = \lim_{x \to 1^+} 1$

$-4 \neq 1$

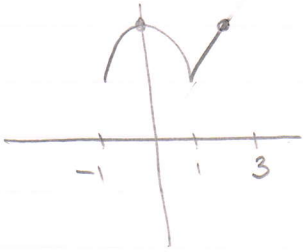
at $\boxed{x = 1}$ $f'(x)$ DNE

$f(0) = 5 - 2(0)^2$

rel max :	(0 , 5)
rel min :	(1 , 3)

$f(1) = 5 - 2(1)^2$

E) $f(x) = \begin{cases} 5 - 2x^2 & -1 \le x \le 1 \\ x + 2 & 1 < x \le 3 \end{cases}$



from D → rel max (0,5) rel min (1,3)

endpoints : $x = -1$ $\quad x = 3$

$f(-1) = 5 - 2(-1)^2 = 3$ $\quad f(3) = (3) + 2 = 5$

*Some textbooks will say these pts are BOTH Absolute + Relative

max : (0,5) ; (3,5)

min : (-1,3) ; (1,3)