

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## 14. Mean Value Theorem

Intermediate Value Theorem  
 Extreme Value Theorem  
 Mean Value Theorem } • Existence theorems  
 } • f must be cts

### Mean Value Theorem

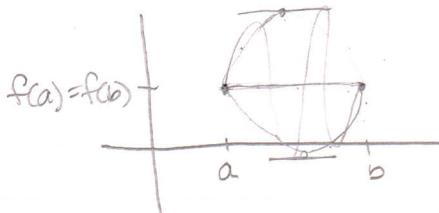
If  $y=f(x)$  is cts on  $[a, b]$ ; and  
 $f$  is differentiable (<sup>cts and slopes everywhere</sup>  
 no cusps/vert tan) on  $(a, b)$

then there exists at least one point  $x=c$  in  $(a, b)$   
 at which

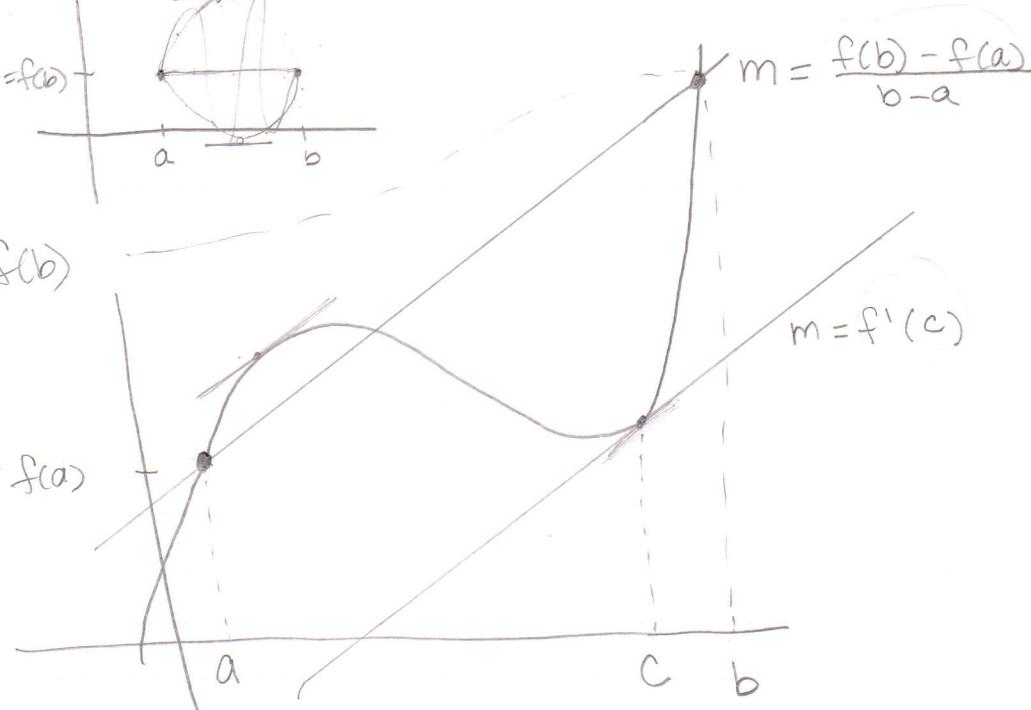
$$\underbrace{f'(c)}_{\text{slope of tangent line @ } x=c} = \frac{\underbrace{f(b) - f(a)}_{\text{slope of secant line through endpoints}}}{b-a}$$

#### \* Rolle's Theorem

→ same as MVT except  $f(a) = f(b)$  e.g. slope = 0



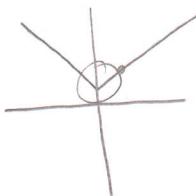
MVT  $f(b)$



Example 1: Explain why the functions do not meet the conditions for the MVT

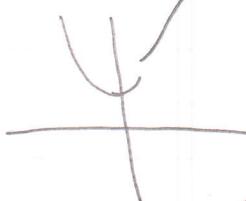
A)  $f(x) = \sqrt{x^2} + 1$

$$|x| = \sqrt{x^2}$$



A has a cusp thus  
f is not differentiable  
at  $x=0$

B)  $f(x) = \begin{cases} x^2 + 3 & x < 1 \\ x^3 + 1 & x \geq 1 \end{cases}$



B is not cts, <sup>at x=1</sup> thus  
the MVT does not apply

Example 2] Find the value(s) of  $c$  that satisfy the MVT OR explain why the MVT is not valid.

A)  $y = x^2 - 4x + 1$  on  $[-1, 1]$

MVT Applies since f is cts and diffable

$$y' = 2x - 4$$

$$y(1) = -2$$

$$y(-1) = (-1)^2 - 4(-1) + 1 = 6$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

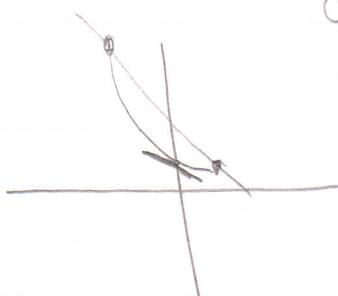
$$2c - 4 = \frac{-2 - 6}{1 - 1} = \frac{-8}{2}$$

$$2c - 4 = -4$$

$$2c = 0$$

$$c = 0$$

← at  $x=0$  the slope of the tangent is  
the same as the slope <sup>through</sup> the end points  
of the secant.



$$y = \sqrt[3]{x} \quad y' = \frac{1}{3\sqrt[3]{x^2}}$$

B)  $y = \frac{x^2 - 1}{2x}$  on  $[-1, 3]$

The MVT does not apply b/c the fn is not cts at  $x=0$

C)  $y = \frac{x^2 - 1}{2x}$   $[1, 3]$

I can use MVT because on  $[1, 3]$  g is cts + differentiable.

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$\frac{c^2 + 1}{2c^2} = \frac{\frac{4}{3} - 0}{3-1} = \frac{4}{3} \div 2 = \frac{4}{3} \times \frac{1}{2},$$

$$\frac{x^2 - 1}{2x} \quad y' = \frac{4x^2 - 2(x^2 - 1)}{(2x)^2}$$

$$y' = \frac{4x^2 - 2x^2 + 2}{4x^2}$$

$$y' = \frac{2x^2 + 2}{4x^2} = \boxed{\frac{x^2 + 1}{2x^2}}$$

$$3c^2 \cdot 3 \cdot \boxed{\frac{c^2 + 1}{2c^2} = \frac{2}{3}} \cdot 3 \cdot 2c^2$$

$$3(c^2 + 1) = 2(2c^2)$$

$$3c^2 + 3 = 4c^2$$

$$\boxed{c = \sqrt{3}}, -\sqrt{3}$$

at  $x = \sqrt{3}$ , the slope of the tan  
is the same as the slope through the endpoints