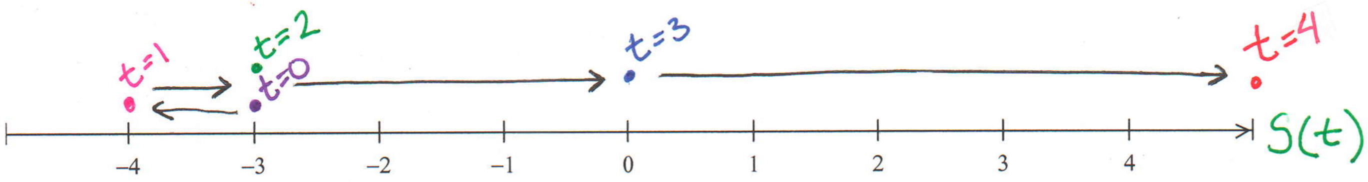
AP Calculus
19 Motion Problems Part 1

Name: P3
Date: ______________________ Per: _____

In AB we discuss straight line motion (BC is 2 dimensional) so either horizontally or vertically. As an object moves, its position is a function of time. For its position function, we will denote the variable $s(t)$, sometimes if the motion is vertical we will use $y(t)$ or horizontal we will use $x(t)$.

Example 1: On the graph below plot the position function $s(t) = t^2 - 2t - 3$ for the values of time $t = 0, 1, 2, 3, 4$. $S(0) = -3$ $S(1) = -4$ $S(2) = -3$ $S(3) = 0$ $S(4) = 5$



When an object moves, its **position changes over time.** So we can say that the velocity function $v(t)$ is the change in position over time. This is a DERIVATIVE.

$$\frac{\Delta d}{\Delta t} \quad \frac{dy}{dx}$$

$$\boxed{v(t) = s'(t)}$$

We define v in the following way:

Motion	$v(t) > 0$	$v(t) < 0$	$v(t) = 0$
Horizontal Line	moving right	moving left	not (at rest) moving
Vertical Line	moving up	moving down	not moving

$$\frac{\Delta d}{\Delta t} = 0$$

$speed = |v(t)|$ – velocity without direction.

The definition of acceleration is the change in velocity over time, again a DERIVATIVE.

$$a(t) = v'(t) = s''(t)$$

So, given the position function, $s(t)$, we can now determine both the velocity and the acceleration function.

Motion	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
Horizontal Line	accelerating to the right	accelerating to the left	Velocity is constant
Vertical Line	accelerating up	accelerating down	Velocity is constant

$$\frac{\Delta V}{\Delta t} = 0$$

Think about acceleration as a "push" in a particular direction.

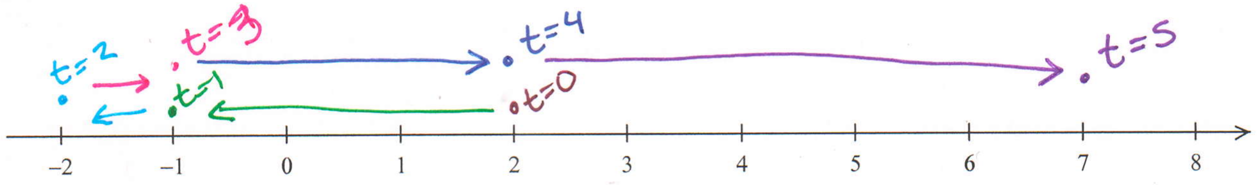
Example 2: Given that a particle is moving along a horizontal line with position function $s(t) = t^2 - 4t + 2$. Find the velocity function and acceleration function for the first 5 seconds, show where the object is on the number line and describe the particles motion.

$S(t) = t^2 - 4t + 2$

$v(t) =$ $2t - 4$

$a(t) =$ 2

t	$s(t)$	$v(t)$	$\lvert v(t)\rvert$	$a(t)$	Description of particle's motion moving L/R Speedup/slowdown	
0	2	-4	4	2	moving L	Slowing down
1	-1	-2	2	2	" "	" "
2	-2	0	0	2	Stopped	about to move right
3	-1	2	2	2	moving R	Speeding up
4	2	4	4	2	" "	" "
5	7	6	6	2	" "	" "



We are generally interesting when the particle is stopped $\boldsymbol{v(t) = s'(t) = 0}$ or when it has no acceleration $\boldsymbol{a(t) = v'(t) = s''(t) = 0}$. We are also interested when the object is speeding up or slowing down. Let's look at the chart to determine the motion possibilities.

	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
$v(t) > 0$	Speeding up (V+a working together)	Speed is decreasing (V+a working against)	Moving right at a constant velocity
$v(t) < 0$	Slowing down	Speeding up	moving L at a constant velocity
Stopped $v(t) = 0$	about to move right	about to move left	not moving

Example 3: A particle is moving along a horizontal line with position function $s(t) = t^2 - 6t + 5$. Do an analysis of the particles direction (right/left), acceleration, motion (speeding up/slowing down), and position.

a. When is the particle stopped ($v(t) = 0$)?

$s'(t) = v(t) = 2t - 6$ $\quad t = 3$

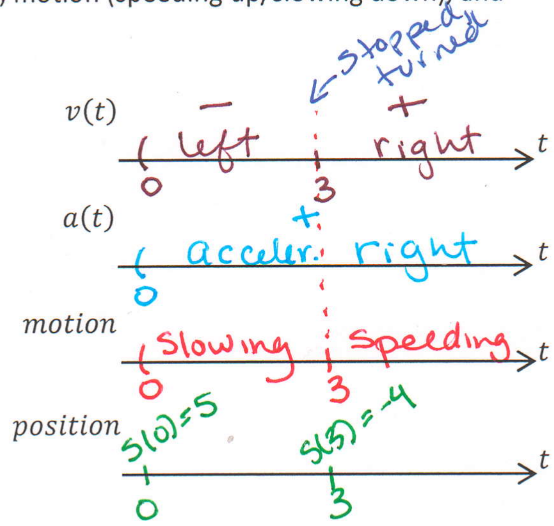
b. When is the particle moving left ($v(t) < 0$) and right ($v(t) > 0$)? Make a sign chart.

c. When is there no acceleration ($a(t) = 0$)?

$s''(t) = v'(t) = a(t) = 2$

d. When is the particle accelerating left ($a(t) < 0$) and right ($a(t) > 0$)? Make a sign chart.

e. Use the chart above to describe the motion and find the position at critical times.

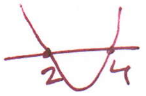


Example 4: A particle is moving along a horizontal line with position function $s(t) = t^3 - 9t^2 + 24t - 4$. Do an analysis of the particles direction, acceleration, motion (speeding up/slowing down) and position.

$s'(t) = v(t) = 3t^2 - 18t + 24$

$0 = 3(t-2)(t-4)$

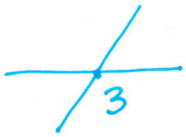
$t = 2, 4$

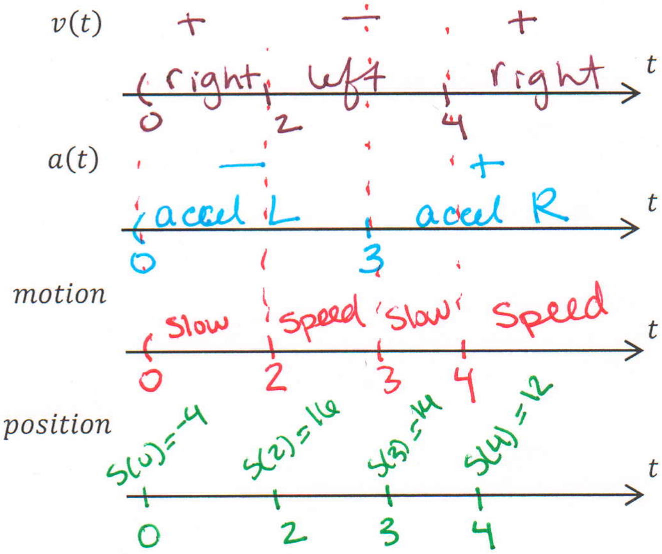


$a(t) = 6t - 18$

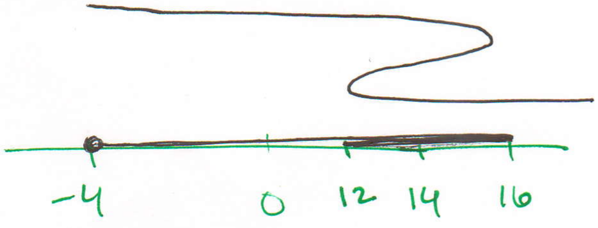
$0 = 6(t-3)$

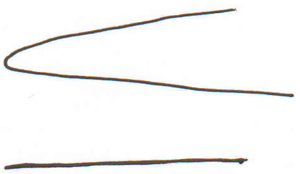
$t = 3$





$s(5) = 16$





AP Released Free Response Motion Problems

Calc

1. a, c, d

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

↳$|v(t)|$ =

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$v(t)=0$ changes sign

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

are v + a working together or against?

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

a) $|v(t)| = 2$ or $\left|-2+(t^2+3t)^{6/5} - t^3\right| = 2$

← on AP test you have to show what you put into the calc

$t = 3.12763, 3.47340$ ← 5 digits past the decimal when you are WORKING

truncated (cutoff) → $t = 3.127$, 3.473 ← 3 digits past decimal for final answer

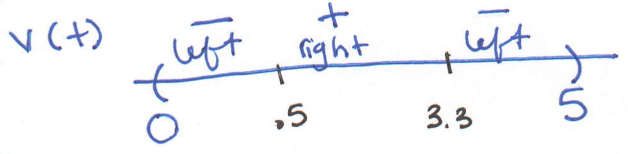
or 3.128 ← rounded

c) $v(t) = 0$

$t = 0.53603, 3.31776$

$t = 0.536, 3.318$ or 3.317

Scratch work NOT Justification



Best 1) the particle changes direction at $t=0.536$ since $v(t)<0$ on $0<t<.536$ and $v(t)>0$ on $.536<t<3.318$ and $t=3.318$ ...

ok 2) changes direction at $t = .536 + 3.317$ since the velocity changes sign at those values

Good 3) Changes at $t=.536$ since velocity goes from neg to positive at $t=3.317$ ....

on test

$v(4) < 0$

$v'(4) = a(4) < 0$

the speed of the particle is increasing at $t=4$ since a and v are

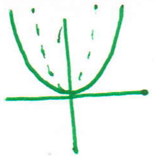
- both negative
- have the same sign
- going the same direction

In calc

$f1(4)$ — #

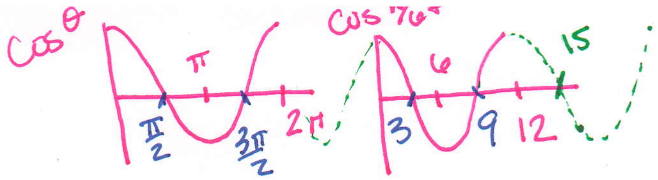
$f2(x) := \frac{d}{dx}(f1(x))$

$f2(4)$ — #



$\frac{\pi}{6}\cdot 0 = 0$

$\frac{\pi}{6}\cdot 12 = 2\pi$



2. a, c non calc

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left? $v(t) < 0$

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

a) $v(t) = 0$

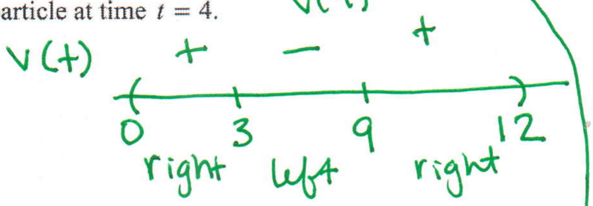
$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ $\quad \theta = \frac{3\pi}{2}$

$\underbrace{\frac{6}{\pi}\cdot\frac{\pi}{6}\cdot t = \frac{\pi}{2}\cdot\frac{6}{\pi}}$ $\quad \frac{6}{\pi}\frac{\pi}{6}t = \frac{3\pi}{2}\cdot\frac{6}{\pi}$

$t = 3$ $\quad t = 9$

$v(4) < 0$



→ the particle is moving to the left

- (3, 9)
- $3 < t < 9$
- from $t = 3$ to $t = 9$

c) $a(t) = v'(t)$

$= -\sin\left(\frac{\pi\cdot t}{6}\right)\cdot\frac{\pi}{6}$

$a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = (-\#)\sin\left(\frac{2\pi}{3}\right)$

$-\cdot + = -$

QII

$\sin > 0$

$a(4) < 0$

Speed of the particle is inc because v + a are going the same direction

3.a Calc

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

are v + a working together or against

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

on test

$v(5.5) < 0$

$v'(5.5) = a(5.5) < 0$

The speed of the particle is increasing at $t = 5.5$ since velocity + acceleration are the same sign

In calc

$f1(x) := 2\sin\left(e^{x/4}\right) + 1$

$f1(5.5) = -0.45337$

$f2(x) := \frac{1}{2}e^{x/4}\cos\left(e^{x/4}\right)$

or

$f2(x) := \frac{d}{dx}(f1(x))$

$f2(5.5) = -1.3585)$