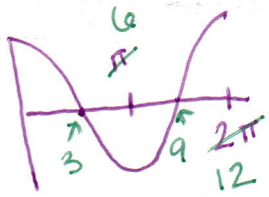

anti ↑ X V a ↓ der

p3

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$\frac{d}{dx}(\sin x) = \cos x$

Question 6

$t=0$, $V(0)=1$, $t=12$, $V(12)=1$

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

$X(0) = -2$ (time, position)

(a) For $0 \le t \le 12$, when is the particle moving to the left? $V<0$

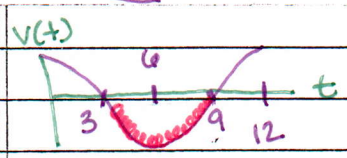
(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$. (absolute value)

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning. (V + a working together?)

(d) Find the position of the particle at time $t = 4$. want #

WTK $V<0$

a)



The particle is moving left
- between $t=3$ + $t=9$
- on the interval $(3,9)$ or $3<t<9$

b) $V \to X$

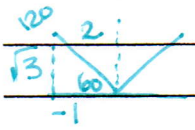
$$\int_0^6 |v(t)|\,dt$$

c) $V \to a$

$V(t) = \cos\left(\frac{\pi}{6}t\right)$ 3x

$a(t) = -\sin\left(\frac{\pi}{6}t\right)\cdot\frac{\pi}{6} = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

S | A
T | C



$V(4) = \cos\left(\frac{2}{3}\pi\right) < 0$ QII

$a(4) = -\frac{\pi}{6}\underbrace{\sin\left(\frac{2}{3}\pi\right)}_{QII\ >0} < 0$

The speed of the particle is increasing at $t=4$ because
→ V + a are working together, → V + a are going the same direction
→ V + a have the same sign.

d) $V \to X$

$$\underbrace{\int_0^4 v(t)\,dt}_{\text{accumulation of distance in first 4 sec}} = \underbrace{X(4)}_{\text{want}} - \underbrace{X(0)}_{-2}$$

position at $t=4$; position at 0 sec

$$\frac{3\sqrt{3}}{\pi} = X(4) + 2$$

$$\boxed{X(4) = \frac{3\sqrt{3}}{\pi} - 2}$$

$$\int_0^4 \cos\left(\overbrace{\frac{\pi}{6}t}^{u}\right)\underbrace{dt}_{\frac{6}{\pi}du}$$

$u = \frac{\pi}{6}t$

$du = \frac{\pi}{6}dt$

$\to dt = \frac{6}{\pi}du$

$u(4) = \frac{2}{3}\pi$

$u(0) = 0$

$$= \frac{6}{\pi}\int_0^{2/3\pi} \cos u\,du$$

$$= \frac{6}{\pi}\left(\sin\left(\frac{2}{3}\pi\right) - \sin(0)\right)$$

$$= \frac{6}{\pi}\left(\frac{\sqrt{3}}{2} - 0\right) = 3\sqrt{3}/\pi$$