

# 29 Differential Equations and Separation of Variables

## Families of Derivatives

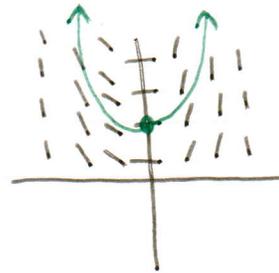
$$y = x^2 + 2$$

$$y = x^2 + C$$

$$y' = 2x$$

general solution

particular solution



Slope Fields

(0, 2)

## Separation of Variables

$$y' = f(x)$$

$$\frac{dy}{dx} = f(x)$$

$$\int dy = \int f(x) dx$$

$$y = F(x) + C$$

1. Separate

2. Integrate

3. Constant

4. Initial Condition

5. Solving for y

can be changed

Example 1: Use Separation of Variables to find  $y=f(x)$ , the particular solution to the given initial value problem.

A)  $y \cdot \frac{dy}{dx} - x = 0$

$$y \frac{dy}{dx} = x$$

$$\int y' dy = \int x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\frac{(-1)^2}{2} = \frac{(-3)^2}{2} + C$$

$$\frac{1}{2} - \frac{9}{2} = C = -\frac{8}{2} = -4$$

$$f(-3) = -1$$

$$y^2 = x^2 + C$$

$$(-1)^2 = (-3)^2 + C$$

$$1 = 9 + C \rightarrow C = -8$$

$$y^2 = x^2 - 8$$

$$y = -\sqrt{x^2 - 8}$$

$$f(-3) = -1$$

$$y^2 = x^2$$

$$y = \pm \sqrt{x}$$

exp  $e^{\#}$

B)  $y' = t \cdot y$   $y(0) = 3$

$$\frac{dy}{dt} = t \cdot y$$

$$\frac{dy}{y} = \int t \cdot dt$$

$$\ln |y| = \frac{t^2}{2} + c$$

$$\log_e a = x \rightarrow e^x = a$$

$$|y| = e^{\frac{t^2}{2} + c} = \underbrace{e^c}_C \cdot e^{\frac{t^2}{2}}$$

$$X^a X^b = X^{a+b}$$

$$\begin{aligned} y &= C e^{\frac{t^2}{2}} \\ 3 &= C e^{0^{\frac{1}{2}}} = C \\ y &= 3 e^{\frac{t^2}{2}} \end{aligned} \quad y(0) = 3$$

C)  $y' + 4xy = 0$   $y(0) = 25$

$$\frac{dy}{dx} = -4xy$$

$$\int \frac{1}{y} dy = \int -4x dx$$

$$\ln |y| = -2x^2 + c$$

$$|y| = e^{-2x^2 + c} = \cancel{e^c} C e^{-2x^2}$$

$$25 = C \underbrace{e^{-2(0)^2}}_1 = C$$

$$y(0) = 25$$

$$y = 25 e^{-2x^2}$$

25 > 0  
OK remove  
abs value

$$D) \quad y^3 \cdot \frac{dy}{dx} - 9x^2 = 0$$

$$y(1) = 2$$

$$y^3 \frac{dy}{dx} = 9x^2$$

$$\int y^3 dy = \int 9x^2 dx$$

$$\frac{y^4}{4} = 3x^3 + C$$

$$\frac{16}{4} = 3(1)^3 + C$$

$$C = 1$$

$$\frac{y^4}{4} = 3x^3 + 1$$

$$y^4 = 12x^3 + 4$$

$$y(1) = 2$$

$$y^4 = 12x^3 + C$$

(1,2)

$$16 = 12 + C$$

$$C = 4$$

$$y^4 = 12x^3 + 4$$

$$y = \sqrt[4]{12x^3 + 4}$$

$$E) \quad \frac{dy}{dx} = -2x - 3$$

$$y(-2) = -1$$

$$\int dy = \int (-2x - 3) dx$$

$$y = -x^2 - 3x + C$$

$$-1 = -(-2)^2 - 3(-2) + C$$

$$-1 = -4 + 6 + C$$

$$C = -3$$

$$y = -x^2 - 3x - 3$$

$$F) \quad \frac{dy}{dx} = 2 \cos x$$

$$y(\pi) = 3$$

$$\int dy = \int 2 \cos x \cdot dx$$

$$y = 2 \sin x + C$$

$$3 = 2 \sin(\pi) + C$$

$$3 = 2 \cdot 0 + C$$

$$C = 3$$

$$y = 2 \sin x + 3$$

$$\sin x \quad -\sin x$$

$$\cos x \quad -\cos x$$



G)

$$\frac{dy}{dx} = \frac{5}{x}$$

$$\int dy = \int \frac{5}{x} dx$$

$$y = 5 \ln|x| + C$$

$$y = 5 \ln(-x) + C$$

$$-2 = 5 \ln(1) + C$$

$$-2 = 5 \cdot 0 + C$$

$$C = -2$$

$$y = 5 \ln(-x) - 2$$

$$y(-1) = -2$$

$x < 0$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\log_e 1 = ? \quad e^? = 1$$

H)

$$\frac{dy}{dx} = \frac{2x}{e^{2y}}$$

$$y(-3) = \frac{\ln(21)}{2}$$

$$\int e^{2y} dy = \int 2x \cdot dx$$

$$\frac{1}{2} e^{2y} = x^2 + C$$

$$\int e^{2y} dy = \int e^u \cdot \frac{1}{2} du$$

$$u = 2y$$

$$du = 2 dy$$

$$\hookrightarrow dy = \frac{1}{2} du$$

$$= \frac{1}{2} \int e^u du$$

$$e^{2y} = 2x^2 + C$$

$$e^{2 \cdot \left(\frac{\ln(21)}{2}\right)} = 2(-3)^2 + C$$

$$e^{\ln 21} = 18 + C$$

$$21 = 18 + C$$

$$C = 3$$

$$e^{2y} = 2x^2 + 3$$

log

$$e^x = a \rightarrow \log_e a = x$$

$$\ln a = x$$

$$\ln(2x^2 + 3) = 2y$$

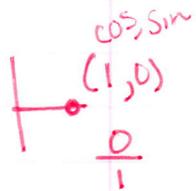
$$y = \frac{1}{2} \ln(2x^2 + 3) = \frac{\ln(2x^2 + 3)}{2} = \ln \sqrt{2x^2 + 3}$$

$$\sin(\theta) = r$$
$$\sin^{-1}(r) = \theta$$

$$I) \quad \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$y(-2) = 0$$

$$\int \sec^2 y \cdot dy = \int 1 dx$$



$$\tan y = x + C$$

$$\tan(0) = -2 + C$$

$$0 = -2 + C$$

$$C = 2$$

$$\tan y = x + 2$$

$$\tan^{-1}(x+2) = y$$

$$y = \tan^{-1}(x+2)$$

$$J) \quad \frac{dy}{dx} = \frac{2x}{y^2}$$

$$y(2) = \sqrt[3]{13}$$

$$\int y^2 dy = \int 2x dx$$

$$\frac{1}{3}y^3 = x^2 + C$$

$$y^3 = 3x^2 + C$$

$$(\sqrt[3]{13})^3 = 3(2)^2 + C$$

$$13 = 12 + C$$

$$C = 1$$

$$y^3 = 3x^2 + 1$$

$$y = \sqrt[3]{3x^2 + 1}$$