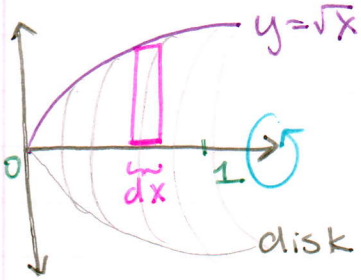
- draw a loop around axis of rotation
- draw rectangle(s) from <u>axis to fn</u>
- Set up integral $\pi\int(\ )^2 d\_$ or $\pi\int(\ )^2-(\ )^2 d\_$ + fill in blanks

<u>Example 1</u>: rotate about the x-axis ($y=0$), bound by $y=\sqrt{x}$ $x=0,1$



radius = axis − function

$\underset{y=}{0} - \underset{y=}{\sqrt{x}}$

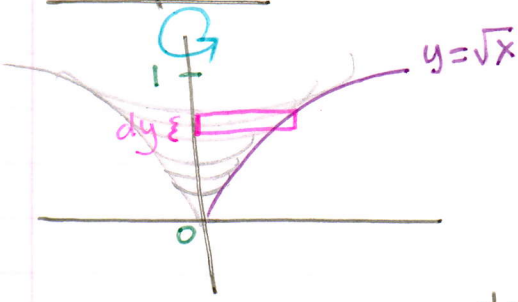
$(0-\sqrt{x})^2 = x$

$(\sqrt{x}-0)^2 = x$

disk → $\boxed{\pi\int_0^1 (0-\sqrt{x})^2\, d\underline{x}}$

<u>Example 2</u>: rotate about the y-axis ($x=0$), bound by $y=\sqrt{x}$ $y=0,1$

$x=y^2$



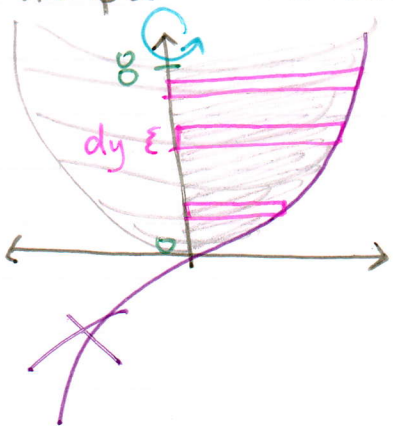
radius = ax − fn

$\underset{x=}{0} - \underset{x=}{y^2}$

disk → $\boxed{\pi\int_0^1 (0-y^2)^2\, d\underline{y}}$

<u>Example 3</u>: rotate about the y-axis ($x=0$) bound by $y=x^3$ $y=0,8$

$x=\sqrt[3]{y}$

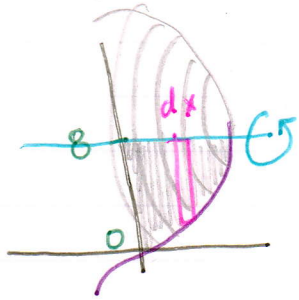


radius = ax − fn

$\underset{x=}{0} - \underset{x=}{\sqrt[3]{y}}$

disk → $\boxed{\pi\int_0^8 (0-\sqrt[3]{y})^2\, d\underline{y}}$

<u>Example 4</u>: rotate about $y=8$ bound by $y=x^3$ $y=0,8$

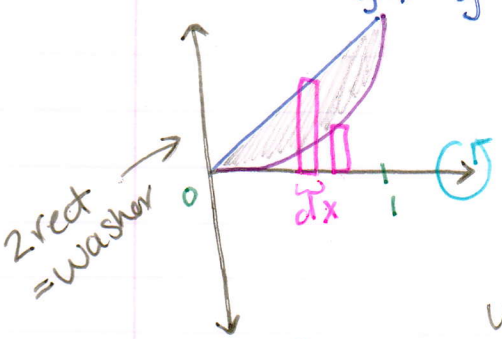


radius = axis − fn

$\underset{y=}{8} - \underset{y=}{x^3}$

disk → $\pi\int_0^8 (8-x^3)^2\, d\underline{x}$

Example 5: Rotate the region bound by $y = x^2$ and $y = x$ about the x-axis $y=0$



Radius = ax - fn

$y=$ $0 - x$ $y=$

radius = ax - fn

$y=$ $0 - x^2$ $y=$

$x^2 = x$

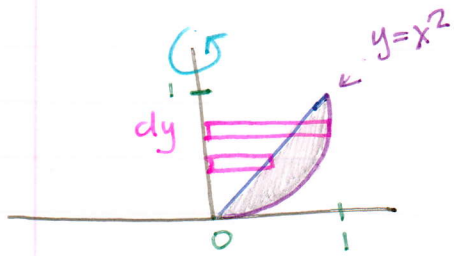
$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

washer → $\boxed{\pi \int_0^1 (0 - x)^2 - (0 - x^2)^2 \, dx}$

Example 6: rotate ex 5 about y-axis $x=0$

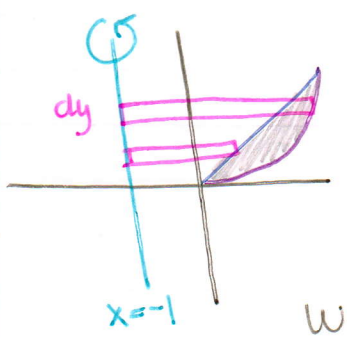


Radius = $0 - \sqrt{y}$ $x=$ $x=$

radius = $0 - y$ $x=$ $x=$

washer → $\pi \int_0^1 (0 - \sqrt{y})^2 - (0 - y)^2 \, dy$

Example 7: rotate ex 5 about $x = -1$

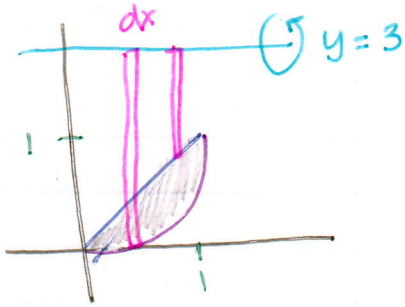


$R = -1 - \sqrt{y}$ $x=$ $x=$

$r = -1 - y$ $x=$ $x=$

Washer → $\boxed{\pi \int_0^1 (-1 - \sqrt{y})^2 - (-1 - y)^2 \, dy}$

Example 8: rotate example 5 about $y=3$

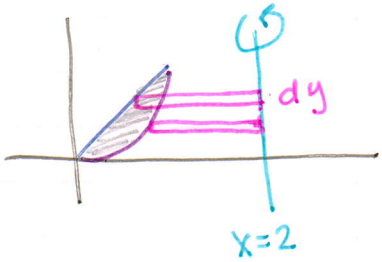


$R = 3 - x^2$ $\quad$ $r = 3 - x$

$y=$ $\quad$ $y=$ $\quad$ $y=$ $\quad$ $y=$

washer $\rightarrow \pi \int_0^1 (3 - x^2)^2 - (3 - x)^2 \, dx$

Example 9: rotate ex5 about $x=2$



$R = 2 - y$ $\quad$ $r = 2 - \sqrt{y}$

$\pi \int_0^1 (2 - y)^2 - (2 - \sqrt{y})^2 \, dy$