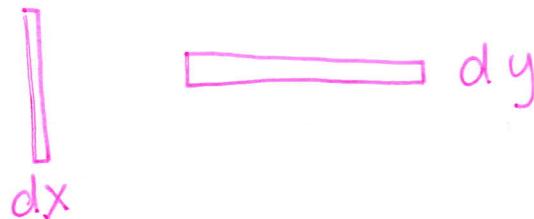


32 Volume: Solids w/known Cross Sections

- Find the intersection of two curves
 - On a graph menu \rightarrow analyze \rightarrow intersection (whole #)
 - On a calc solve ($f(x) = g(x)$, x)
 - Find distance/length between two curves
 - top - bottom dx
 - Right - Left dy
 - furthest - closest
 - Be aware of how many \int are needed
 - new top/bottom fn
- What shape will the cross section be?
- Square = s^2
 - rectangle = (base)(height)
 - equilateral $\Delta = \frac{\sqrt{3}}{4} s^2$
 - Semi circle / quarter circle = $\frac{1}{2}$ or $\frac{1}{4} \pi r^2$
* What is the radius?
- Volume = \int Area

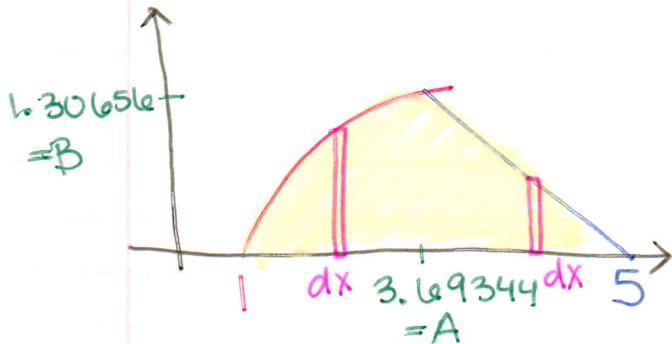
DRAW RECTANGLES!!



$$y = \ln(x) \quad y = 5 - x$$

$$x = e^y \quad x = 5 - y$$

Example 1: Consider the region R in the first quadrant bound by the x-axis, $y = \ln(x)$, and $y = 5 - x$



a) Find the area of Region R.

$$dx \rightarrow \int_1^A \ln(x) dx + \int_A^5 5-x dx$$

$$dy \rightarrow \int_0^B ((5-y) - e^y) dy$$

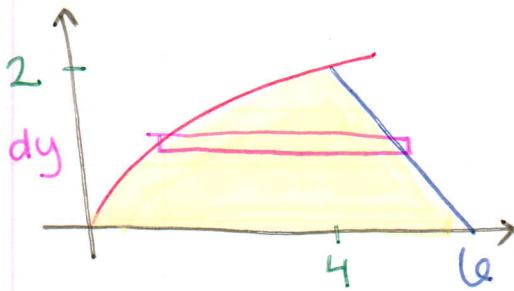
b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$$A(x) = (\underbrace{\ln(x)}_{\text{Side}})^2 \quad \text{and} \quad (\underbrace{5-x}_{\text{Side}})^2$$

$$\text{Volume} = \int_1^A (\ln(x))^2 dx + \int_A^5 (5-x)^2 dx$$

$$\begin{array}{ll} y = \sqrt{x} & y = 6-x \\ x = y^2 & x = 6-y \end{array}$$

Example 2: let R be the region bound by the x -axis, $f(x) = \sqrt{x}$, and $g(x) = 6-x$



a) Find the area of Region R

$$\int_0^2 (6-y) - y^2 dy$$

$$\int_0^4 \sqrt{x} dx + \int_4^6 (6-x) dx$$

b) The region R is the base of a solid. For each y , $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$.

$$\text{Rectangle } A(y) = (\text{base})(\text{height}) = (6-y-y^2)(2y) = 12y - 2y^2 - 2y^3$$

$$\text{Volume} = \int_0^2 12y - 2y^2 - 2y^3 dy$$

$$= \left[6y^2 - \frac{2y^3}{3} - \frac{y^4}{2} \right]_0^2$$

$$= (6 \cdot 4) - \frac{16}{3} - 8$$

$$= 24 - 8 - \frac{16}{3}$$

$$24 - 8 = 16$$

$$16 \cdot 3 = 48$$

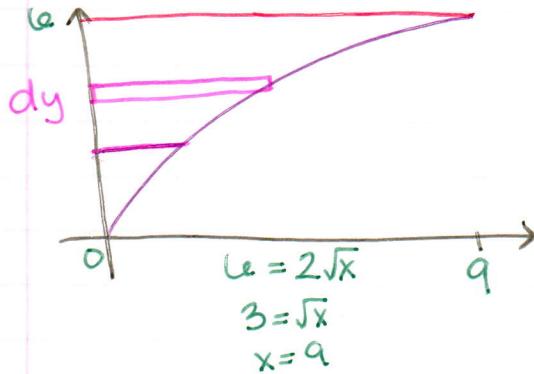
$$= \frac{48}{3} - \frac{16}{3}$$

$$= \boxed{\frac{32}{3}}$$

$$y = 2\sqrt{x}$$

$$\left(\frac{y}{2}\right)^2 = x$$

Example 3: let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$ and the horizontal line $y=6$ and the y -axis.



a) Find the area of region R

$$\int_0^6 \left(\frac{y}{2}\right)^2 dy$$

$$\int_0^6 (6 - 2\sqrt{x}) dx$$

b) Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in Region R . Write but do not evaluate an integral expression that gives the volume of the solid.

$$\text{Rectangle} = (\underbrace{\left(\frac{y}{2}\right)^2}_{\text{base}}) \times (\underbrace{3 \cdot b}_{\text{height}}) = \underbrace{\left(\frac{y}{2}\right)^2}_{\text{base}} \cdot 3 \underbrace{\left(\frac{y}{2}\right)^2}_{\text{height} = 3 \cdot b}$$

$$\text{Volume} = \int_0^6 \underbrace{\left(\frac{y}{2}\right)^2 \cdot 3 \cdot \left(\frac{y}{2}\right)^2}_{\text{rectangle}} dy$$

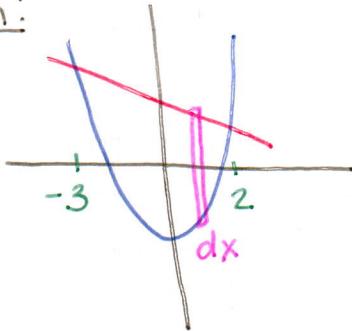
\nwarrow prism figure
 $\rightarrow 3d$ figure

Example 4: Consider the region bound by the graphs

$$y = x^2 - 3 \text{ and } y = -x + 3$$

Find the volume of the solid created using cross sections perpendicular to the x-axis in the shape of:

graph:



bounds:

$$x^2 - 3 = -x + 3$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3, 2$$

length of rect:

$$-x + 3 - (x^2 - 3)$$

top - bottom

$$-x + 3 - x^2 + 3$$

$$= \underbrace{-x^2 - x + 6}_{\text{base}/\text{side}}$$

a) Squares = $\underset{\text{side}}{(-x^2 - x + 6)^2}$

$$\text{Volume} = \int_{-3}^2 (-x^2 - x + 6)^2 dx$$

b) equilateral triangle = $\frac{\sqrt{3}}{4} \underset{\text{side}}{(-x^2 - x + 6)^2}$

$$\text{Volume} = \frac{\sqrt{3}}{4} \int_{-3}^2 (-x^2 - x + 6)^2 dx$$

c) rectangle w/ height $(2x+1)$ - $\underset{\text{base}}{(-x^2 - x + 6)} \underset{\text{height}}{(2x+1)}$

$$V = \int_{-3}^{-2} (-x^2 - x + 6)(2x+1) dx$$

d) semi circle whose diameter is in the base = $\frac{1}{2} \cdot \pi \left(\frac{-x^2 - x + 6}{2} \right)^2$

$$\frac{\pi}{8} \int_{-3}^2 (-x^2 - x + 6)^2 dx$$

$$\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$$

$$\frac{1}{2 \cdot 2 \cdot 2}$$