3 Limits: Analytical Approach

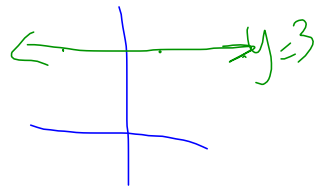
Basic Limit Laws:

assume $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist

Sum Law:

$$\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$$

Constant Multiple Law:



$$\lim_{x\to c} k\, f(x) = k \lim_{x\to c} f(x)$$

$\lim_{x\to 0} 3 = 3$

Product Law:

$$\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \lim_{x\to c} g(x)$$

Quotient Law:

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

$\lim_{x\to c} g(x) \neq 0$

Powers and Roots:

$$\lim_{x\to c} f(x)^{p/q} = \left(\lim_{x\to c} f(x)\right)^{p/q}$$

Steps to evaluate limits

1) PLUG IN

can't plug in (indeterminate?)

2) factor/rationalize/multiply by the conjugate/common denominator etc

THEN REDUCE and

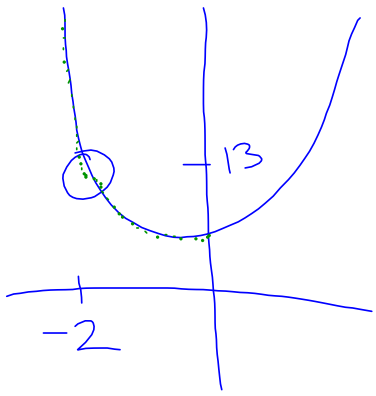
PLUG IN!!

Example 1) $\lim_{x \to -2} x^2 - 4x + 1 = 13$

$= (-2)^2 - 4(-2) + 1$

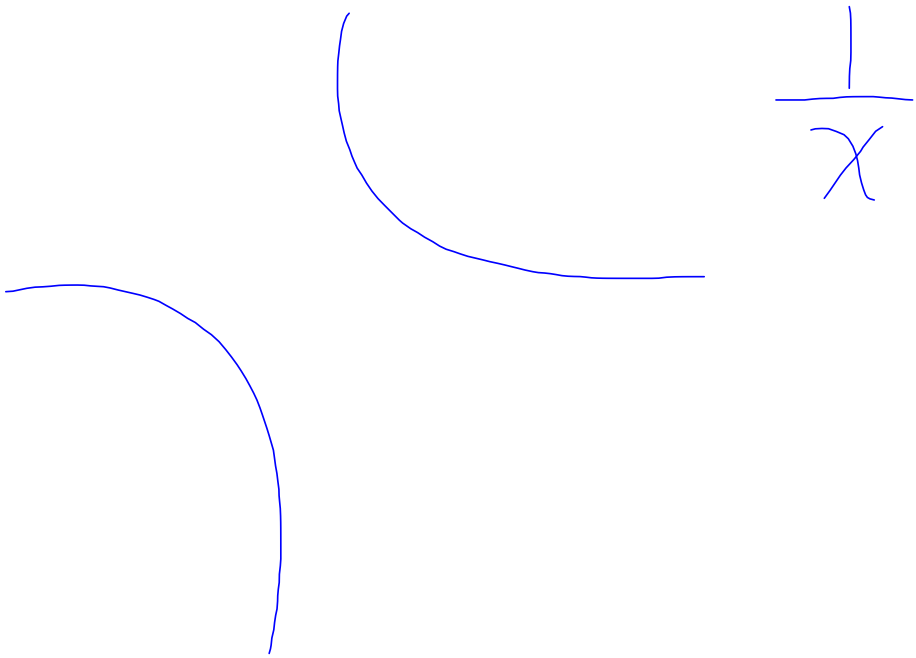
$= 4 + 8 + 1$

$= 13$

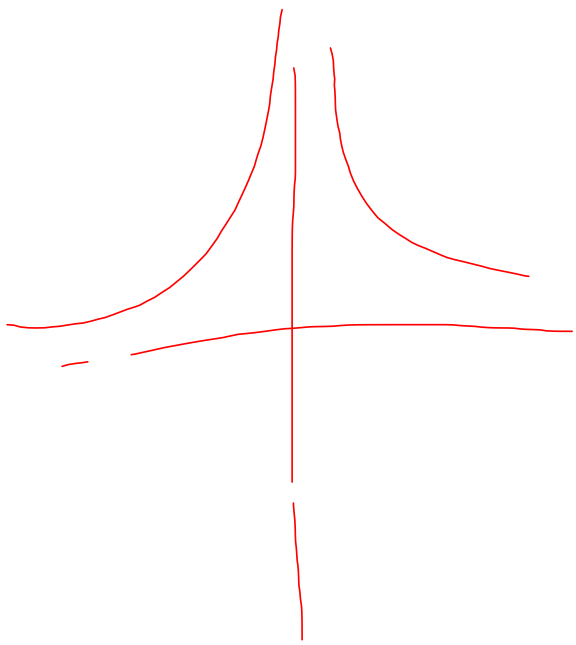


$$\left(\lim_{x \to -2} x\right)^2 - 4 \lim_{x \to -2} x + \lim_{x \to -2} 1$$

Example 2) $\lim_{x \to -2} \frac{2x-6}{x-2} = \frac{5}{2}$



Example 3) $\lim_{x \to 3} x^{-2} = \frac{1}{9}$



$y = \frac{1}{x^2}$

Example 4)

$$\lim_{q \to -1} q^4 - 8q^3 + 9q^2 - 12q + 13 =$$

$$= 1 + 8 + 9 + 12 + 13 = 43$$

Example 5) $\lim_{x \to 2} \frac{x-2}{3x-4} = \frac{0}{2} = 0$

$= \frac{(2)-2}{3(2)-4}$

$(2,0)$

$3x-4=0$

$x=4/3$

Example 6) $$\lim_{x \to 2} \frac{3x-4}{x-2} = \frac{2}{0} = \text{DNE}$$

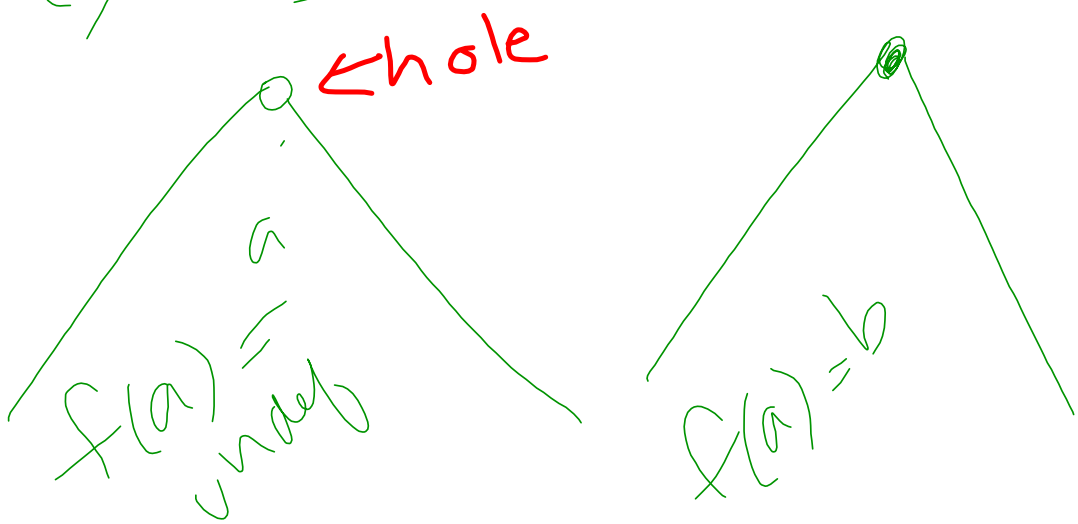
*we will deal more exclusively with these types of limits in the next section

Example 7)

$$\lim_{x\to 3}\frac{5x-15}{x-3} = \frac{0}{0} =$$

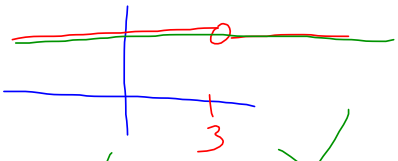
The funcon $f(x) = \frac{5x-15}{x-3}$ and $g(x) = 5$ agree everywhere except at $x = 3$, where f has a hole at $x = 3$ and g is connuous.

$$\lim_{x\to 3}\frac{5\cancel{(x-3)}}{\cancel{(x-3)}} = 5$$

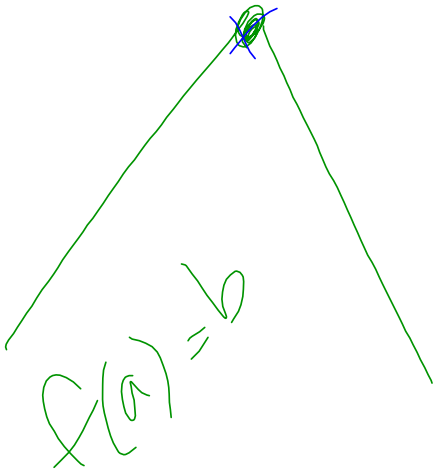


Example 7)

$$\lim_{x\to 3}\frac{5x-15}{x-3} = \lim_{x\to 3} 5 = 5$$



$$\frac{5\cancel{(x-3)}}{\cancel{(x-3)}} = 5$$



Example 8)

HA $y = 1$

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 + x - 12} = \frac{2}{7}$$

$(x-3)(x+4)$

$$\lim_{x \to 3} \frac{x-1}{\underbrace{x+4}} = \frac{2}{7}$$

VA $x = -4$

$\lim_{x \to -4} f(x) = \text{Undef}$

Example 9)

$$\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4}$$

$(x+2)(x-4)$

$(x+2)(x-2)$

$x = 2, -2$

hole

$$\lim_{x \to -2} \frac{x-4}{x-2} = \frac{3}{2}$$

$$\lim_{x \to 2} \frac{x-4}{x-2} = \frac{-2}{0}$$

Example 10)

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

$(x+4)(x-4)$

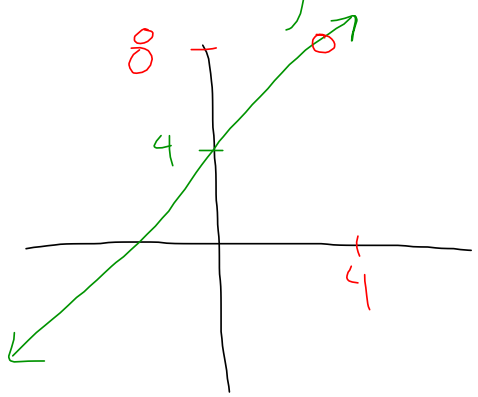
$x = 4$

hole

$$\lim_{x \to 4} \boxed{x + 4} = 8$$

$$\begin{array}{c|c} 1 & x^2 \\ \hline 0 & x^2 \end{array}$$

$y = x + 4$



$x - 4 = 0$

$x = 4$

3 Limits: Analytical Approach

$$y = \frac{5x - 15}{x - 3} \xrightarrow{y=} 5$$

$$y = \frac{x^2 - 16}{x - 4} \xrightarrow{y=} x + 4$$

Example 11)

$$\lim_{x \to 3} \frac{x^2 + 2x - 3}{x^2 + 6x + 9} = \frac{1}{3}$$

$(x+3)(x-1)$

$(x+3)(x+3)$

$x =$

Example 11)

$$\lim_{x\to 3}\frac{x^2+2x-3}{x^2+6x+9}$$

$(x+3)(x-1)$

$(x+3)(x+3)$

hole

VA

$$\lim_{x\to 3}\frac{(x-1)}{(x+3)}$$

$$=\frac{-4}{0}$$

The notes from the sub will be posted in a separate link.

ex 12-18

$-3x^2 - 2ax + a^2$

Example 19) $$\lim_{x \to a} \frac{(x+a)^2 - 4x^2}{x-a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(-3x-a)}{\cancel{(x-a)}}$$

$$\begin{array}{r} 3ax \\ -ax \\ \hline 2ax \end{array}$$

$$\lim_{x \to a} (-3x - a) = \boxed{-4a}$$

$-3a - a$

Example 20)

$$\lim_{h\to 0}\frac{\frac{1}{(h+2)^2}-\frac{1}{4}}{h}$$

$(h+2)^2$
$= h^2 \times 4h \times 4$

$$\lim_{h\to 0}\frac{1}{h}\left(\frac{4-(h+2)^2}{4(h+2)^2}\right)$$

$$\lim_{h\to 0}\frac{1}{h}\left(\frac{4-h^2-4h-4}{4(h+2)^2}\right)$$

$$\lim_{h\to 0}\frac{\cancel{h}(-h-4)}{4\cancel{h}(h+2)^2}=\frac{-4}{16}=\boxed{\frac{-1}{4}}$$

Special Trig Limits you need to MEMORIZE!!

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0$$

Example 21)

$$\lim_{x \to 0} \frac{\sin x}{8x}$$

$$\frac{1}{8} \underbrace{\lim_{x \to 0} \frac{\sin x}{x}}_{\frac{1}{8} \cdot 1} = \frac{1}{8}$$

Example 22)

$$\lim_{x \to 0} \frac{\sin 8x}{8x} \cdot 8$$

$$= 8 \cdot \lim_{x \to 0} \frac{\sin(8x)}{(8x)}$$

$$= 8 \cdot 1$$

$$= 8$$

Example 23) $$\lim_{x \to 0} \frac{5(1-\cos x)}{x}$$

$$5 \cdot \lim_{x \to 0} \frac{(1-\cos x)}{x}$$

$$= 5 \cdot 0$$

$$= 0$$

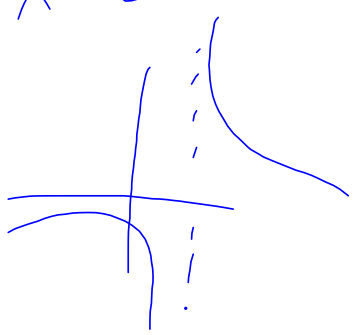
Example 24) $\lim_{x \to 0} \frac{\sin x(1-\cos x)}{x^2}$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{(1-\cos x)}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{(1-\cos x)}{x} = 0$$

$$1 \cdot 0$$

Assignment #3 Due:

Wed 8/14

$$\lim_{x \to 2} \frac{1}{x-2} = \frac{\#}{0}$$