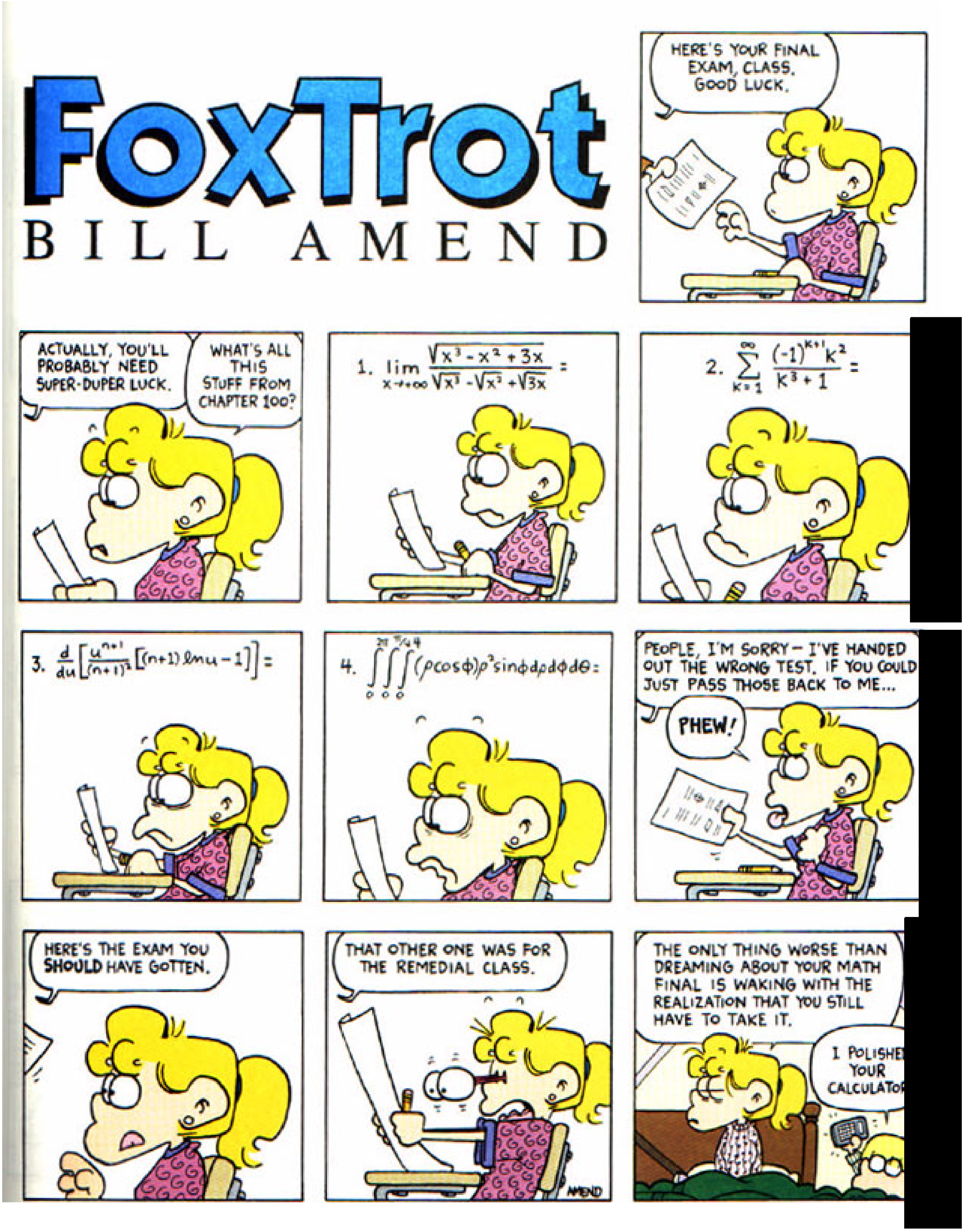
6 Definition of Derivative and Differentiability



6 Definition of the Derivative and Differentiability

$$m_{tan} \approx \frac{f\left(x+h\right)-f(x)}{h}$$

$$m_{tan} = \lim_{h\to 0}\frac{f\left(x+h\right)-f(x)}{h}$$

$$f'\left(x\right) = \lim_{h\to 0}\frac{f\left(x+h\right)-f(x)}{h}$$

read: "f prime of x"

First Derivative	y'	f'(x)	$\frac{dy}{dx}$ $\frac{\Delta y}{\Delta x}$	$\frac{d}{dx}f(x)$
Second Derivative	y''	f''(x)	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}f(x)$
Third Derivative	y'''	f'''(x)	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}f(x)$
Fourth Derivative	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}f(x)$
nth Derivative	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}f(x)$

y prime, y double prime, y triple prime

f prim of x, f double prime of x

dy dx, the derivative of y with respect to x

d dx of f at x

remember all that work you had to do with limits??

$$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$$

$$\frac{\left(\frac{1}{x+4} - \frac{1}{4}\right)}{x}\left(\frac{4(x+4)}{4(x+4)}\right) = \frac{4-(x+4)}{x(4(x+4))} = \frac{4-x-4}{x(4(x+4)}$$

$$= \frac{-x}{x(4)(x+4)} = \frac{-1}{4(x+4)}$$

$$\lim_{x\to 0} \frac{-1}{4(x+4)} = \frac{-1}{4(0+4)} = -\frac{1}{16}$$

$$\lim_{\Delta x\to 0} \frac{2(x+\Delta x)-2x}{\Delta x} = \lim_{\Delta x\to 0} \frac{\cancel{2x}+2\Delta x-\cancel{2x}}{\Delta x} = \lim_{\Delta x\to 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x\to 0} 2 = 2$$

$$\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x}$$

$$\frac{(x+\Delta x)^2-x^2}{\Delta x} = \frac{\cancel{x^2}+2x(\Delta x)+(\Delta x)^2-\cancel{x^2}}{\Delta x} = \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}} = 2x+\Delta x$$

$$\lim_{\Delta x\to 0} 2x+\Delta x = 2x+0 = 2x$$

you were secretly finding derivatives!

What are the three reasons a limit would not exist?

1) left and right side do not agree

2) unbounded behavior

3) OSCILLATING

A derivative is the

SLOPE OF THE TANGENT LINE

you should know this like you know...

a limit is a ... **Y-VALUE**

An alternate form of the derivative is

$$f'(a) = \lim_{x \to a} \boxed{\frac{f(x) - f(a)}{x - a}}$$

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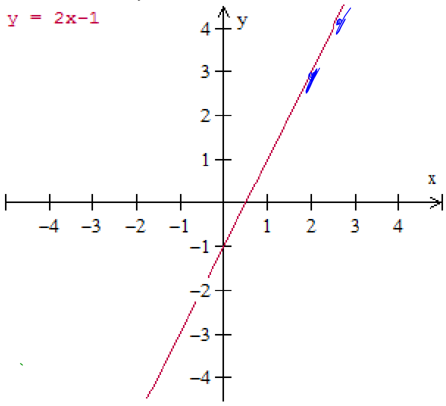
Example 1: Use the graph to determine (or estimate) the derivative at the given point



$f(2) = 3 \leftarrow y$

$f'(2) = 2 \leftarrow m$

$f'(3) = 2$

"what is the slope at $x = 3$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{(2x - 1) - (2a - 1)}{x - a}$$

$$= \lim_{x \to a} \frac{2(\cancel{x - a})}{(\cancel{x - a})}$$

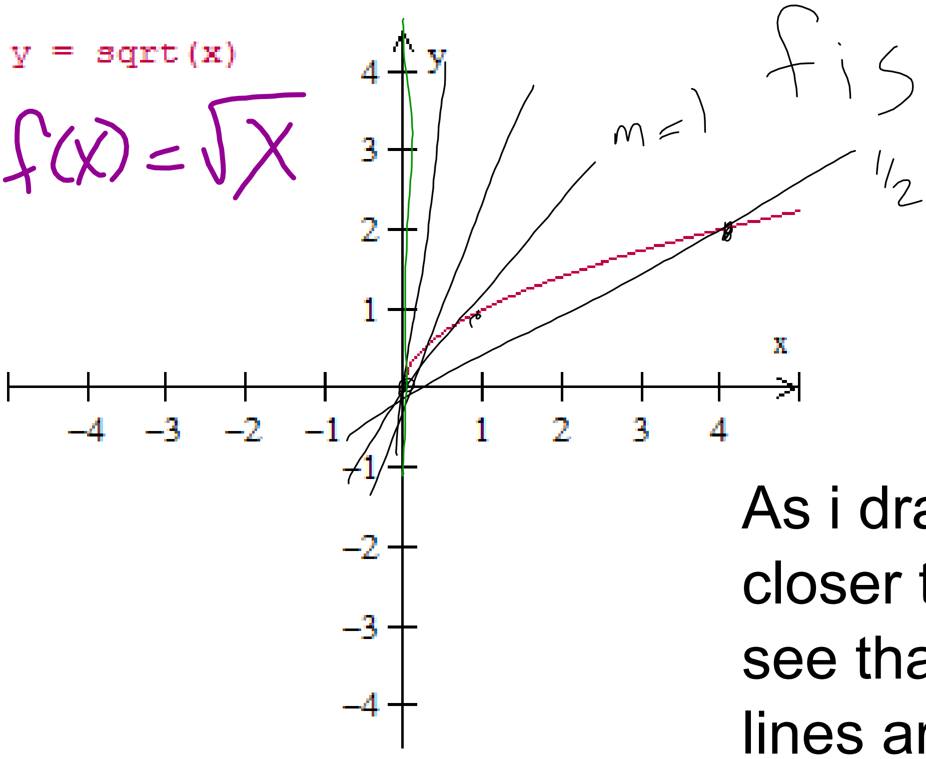
$$= \lim_{x \to a} 2 = 2$$

$$\boxed{f'(a) = 2}$$

This function is everywhere differentiable.

$f(x) = \sqrt{x}$

f is cts on $[0, \infty)$



As i draw slopes closer to x=0, we see that the secant lines are becoming more and more VERTICAL, therefor this function is NOT differentiable at x=0.

$\frac{\#}{0}, \frac{0}{0}, \frac{0}{\#}$

$f'(0) = \text{undef}$

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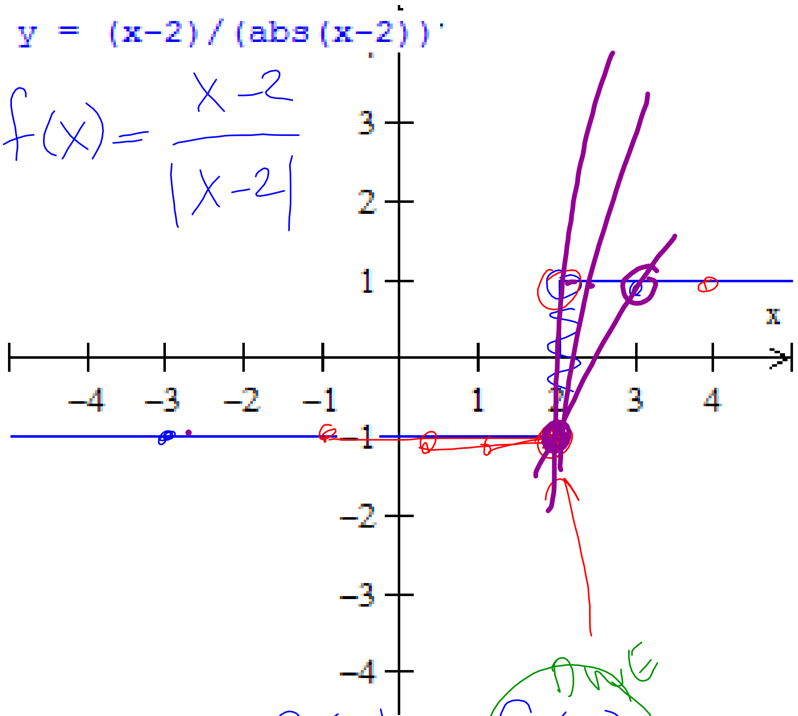

$$f(x)=\frac{x-2}{|x-2|}$$

$$f'(-3)=0$$

$$f'(3)=0$$

$$\lim_{x\to 2^-}\frac{f(x)-\overset{\text{DNE}}{f(2)}}{x-2}$$

$$\approx \frac{f(1)-f(2)}{x-2}=0$$

$$\lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2}=\frac{\#}{0}=\text{undef}$$

This function is not differentiable at x=2

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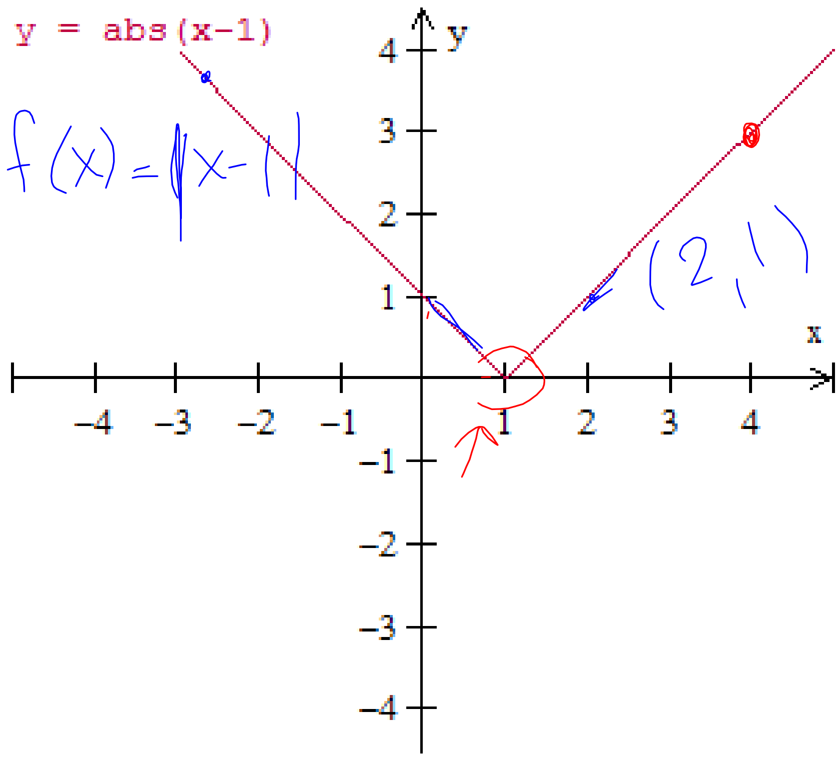

$f'(2) = 1$

$f'(-3) = -1$

what is the slope of f at x=-3

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} \approx \frac{f(0) - f(1)}{0-1} = -1$$

$f'(1) = DNE$

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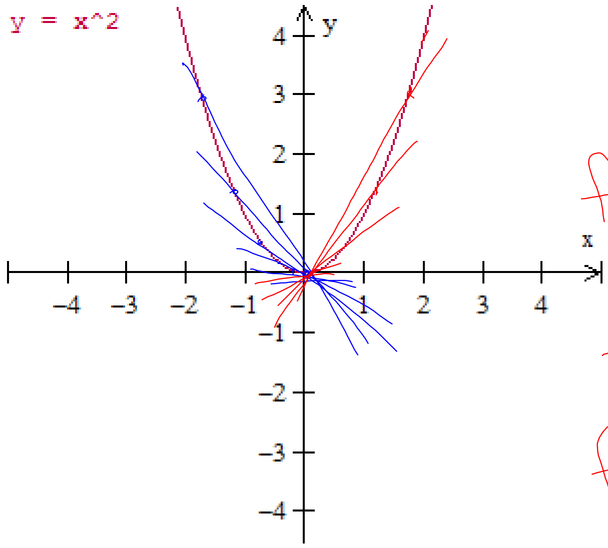

$f'(1) = 2$

$f(0) = 0$

$f'(0) = 0$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} x + 1$$

$$= \boxed{2}$$

y value

$$\underbrace{f'(-3)}_{\text{slope}} = \lim_{x \to -3} \frac{f(x) - f(-3)}{x + 3}$$

$$= \lim_{x \to -3} \frac{x^2 - 9}{x + 3}$$

$$= \lim_{x \to -3} x - 3 = -6$$

the three reasons a DERIVATIVE would fail to exist

1) any type of discontinuity

(no point $\Rightarrow$ no tangent)

2) a corner or cusp, (not a smooth change in the tangent line

3) Vertical tangent (slope=undefined, ∞)

Differentiable

$\rightarrow$ Cts + slope everywhere

Assignment #6 due:

Fri 9/13