

## 7. Rules for Derivatives

Ex 1  $\frac{d}{dx}[c] = 0$  Constant Rule

$$y = 9$$

$$y' = 0$$

$$\frac{dy}{dx} = 0$$

$$f(x) = \sqrt{7}$$

$$f'(x) = 0$$

$$\frac{d}{dx}[f(x)] = 0$$

$$s(t) = \pi$$

$$s'(t) = 0$$

$$\frac{ds}{dt} = 0$$

Ex 2  $\frac{d}{dx}(x) = 1$

$$y = x$$

$$y' = 1$$

$$w = x$$

$$w' = 1$$

$$\frac{dw}{dx} = 1$$

$$\frac{d}{dx}[w] = 1$$

Ex 3  $\frac{d}{dx}[x^n] = n \cdot x^{n-1}$  Power Rule

$$y = x^2$$

$$y' = 2x^1$$

$$f(x) = x^{10}$$

$$f'(x) = 10x^9$$

$$s(t) = \sqrt{t} = t^{1/2}$$

$$\frac{ds}{dt} = \frac{1}{2} \cdot t^{-1/2} = \frac{1}{2\sqrt{t}}$$

$$g(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$$

$$g'(x) = \frac{-1}{3} x^{-4/3} = \frac{-1}{3 x^{4/3}} = \frac{-1}{3 \sqrt[3]{x^4}}$$

$-\frac{1}{3} - \frac{3}{3}$

$$\text{Ex 4} \quad \frac{d}{dx} [c \cdot f(x)] = c \cdot f'(x)$$

Constant multiple rule

$$y = 3x^7$$

$$y' = 3(7x^6) = 21x^6$$

$$f(x) = 4x^3$$

$$f'(x) = 12x^2$$

$$S(t) = 5x^7$$

$$S'(t) = 5 \cdot 7x^6 = 35x^6$$

$$\text{Ex 5} \quad \frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Sum/difference rule

$$y = 5x^2 - 3x + 7$$

$$y' = 10x - 3 + 0$$

$$x^0 = 1$$

$$y = 9(5x^2 - 3x + 7)$$

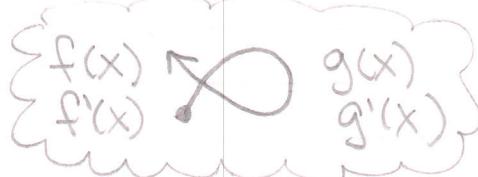
$$y' = 9(10x - 3)$$

$$f(x) = 5x^3 - 7x^2 + 3x + 1$$

$$f'(x) = 15x^2 - 14x + 3$$

$$\text{Ex 6} \quad \frac{d}{dx} [f(x) \cdot g(x)] = f' \cdot g + g' \cdot f$$

Product Rule



$$f(x) = (3x - 7)(4x^3 + 3x - 1)$$

$$\begin{array}{r} 3x - 7 \\ \times 4x^3 + 3x - 1 \\ \hline 12x^2 + 3 \end{array}$$

$$f'(x) = (3)(4x^3 + 3x - 1) + (12x^2 + 3)(3x - 7)$$

$$\underbrace{3x}_{f'(x)} \underbrace{(7x^2 + 3x + 2)}_{g(x)}$$

(6 cont)

Product Rule

$$\frac{d}{dx}(f \cdot g) = f' \cdot g + g' \cdot f$$

$$f' \otimes g$$

$$f(x) = (\underbrace{\sqrt{x} + 3x^7 - 4}_{f(x)}) (\underbrace{x^{5/3} - (6 + x^5)}_{g(x)})$$

$x^{1/2}$

$$\frac{\sqrt{x} + 3x^7 - 4}{2\sqrt{x}} + 21x^6$$

$$\frac{5}{3}x^{2/3} - (6 + x^5) + 5x^4$$

$$f'(x) = \left( \frac{1}{2\sqrt{x}} + 21x^6 \right) (x^{5/3} - (6 + x^5)) + \left( \frac{5}{3}x^{2/3} + 5x^4 \right) (\sqrt{x} + 3x^7 - 4)$$

$$\text{Ex 7} \quad \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f' \cdot g - g' \cdot f}{g^2}$$

$$f' \otimes \frac{g}{g'}$$

$$h(x) = \frac{(\sqrt{x} + 3x^7 - 4)}{(x^{5/3} - (6 + x^5))}$$

$$h'(x) = \frac{\left( \frac{1}{2\sqrt{x}} + 21x^6 \right) (x^{5/3} - (6 + x^5)) - \left( \frac{5}{3}x^{2/3} + 5x^4 \right) (\sqrt{x} + 3x^7 - 4)}{(x^{5/3} - (6 + x^5))^2}$$

$$g(x) = \frac{5x+2}{x^2-1}$$

$$5x+2 \otimes \frac{x^2-1}{2x}$$

$$g'(x) = \frac{(5)(x^2-1) - (2x)(5x+2)}{(x^2-1)^2}$$

$$\textcircled{11}(x) = \frac{5x+3}{x^2+4x-2}$$

$$\textcircled{11}'(x) = \frac{(5)(x^2+4x-2) - (2x+4)(5x+3)}{(x^2+4x-2)^2}$$

$$2 \cancel{x} \frac{\sin x}{\cos x}$$

Ex 8 | Find each of the Derivatives

A)  $f(x) = 3x^2 + (2)\sin x - \ln x$

$$f'(x) = (6x) + 2\cos x - \frac{1}{x}$$

$$2\sin x = \sin x + \sin x$$

$$2\cos x = \cos x + \cos x$$

B)  $g(x) = 4x^3 - \cos x + \ln x - 6$

$$g'(x) = 12x^2 + \sin x + \frac{1}{x}$$

C)  $h(x) = (3x^2 - \sin x)(\ln x + 5^x)$

$$\begin{matrix} 5y \\ y \cdot 5 \end{matrix}$$

$$\begin{array}{c} 3x^2 - \sin x \\ 6x - \cos x \end{array} \cancel{\times} \quad \begin{array}{c} \ln x + 5^x \\ \frac{1}{x} + 5^x (\ln 5) \end{array}$$

$$h'(x) = (6x - \cos x)(\ln x + 5^x) + \left(\frac{1}{x} + 5^x \ln 5\right)(3x^2 - \sin x)$$

d)  $y = \frac{\tan x}{3^x \cdot e^x}$

$$(a^4 b^4) = (ab)^4$$

$$\begin{array}{c} \tan x \\ \sec^2 x \end{array} \cancel{\times} \quad \begin{array}{c} 3^x (e^x) \\ (3^x \ln 3)(e^x) + (e^x)(3^x) \\ \boxed{3^x e^x \cdot \ln 3 + 3^x e^x} \end{array}$$

$$\begin{array}{c} 3^x \\ 3^x \ln 3 \end{array} \cancel{\times} \quad \begin{array}{c} e^x \\ e^x \end{array}$$

$$y' = \frac{(\sec^2 x)(3^x e^x) - (3^x e^x \ln 3 + 3^x e^x)(\tan x)}{(3^x \cdot e^x)^2}$$

$$\boxed{3^x e^x = (3e)^x}$$

$$y = \frac{\tan x}{(3e)^x}$$

$$\begin{array}{c} \tan x \\ \sec^2 x \end{array} \cancel{\times} \quad \begin{array}{c} (3e)^x \\ (3e)^x \ln 3e \end{array}$$

$$y' = \frac{\sec^2 x (3e)^x - ((3e)^x) \ln 3e \cdot \tan x}{((3e)^x)^2}$$

$$\frac{ab - bc}{b^2}$$

$$y' = \frac{\sec^2 x - \ln 3e \cdot \tan x}{(3e)^x}$$

$$e) g(x) = \frac{\tan x - \csc x}{3x^2 + 5x + \log_4 x}$$

$$\frac{\tan x - \csc x}{\sec^2 x + \csc x \cot x} \neq \frac{3x^2 + 5x + \log_4 x}{6x + 5 + \frac{1}{x \cdot \ln 4}}$$

$$g'(x) = \frac{(\sec^2 x + \csc x \cot x)(3x^2 + 5x + \log_4 x) - (6x + 5 + \frac{1}{x \cdot \ln 4})(\tan x - \csc x)}{(3x^2 + 5x + \log_4 x)^2}$$

$$f) h(x) = \frac{\sin x}{\cos x} \quad \frac{\sin x}{\cos x} \neq \frac{\cos x}{-\sin(x)}$$

$$h'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Ex 9) Find the value of  $x$  where  $m=?$

A)  $f(x) = x^2 + 2x - 7$   
 $f'(x) = \cancel{2x} + 2$   
 $0 = 2x + 2$   
 $x = -1$

$m=0$   
 $f'(?)=0$

at  $x=-1$ , the slope of  $f$  is 0

$x$	$f'(x)$
-3	$2(-3) + 2 = -4$
-2	-2
-1	0
0	2
1	4

B)  $f(x) = x^3 + 6x^2 + 12x - 2$   
 $f'(x) = 3x^2 + 12x + 12$   
 $f'(x) = 3(x^2 + 4x + 4)$   
 $0 = 3(x+2)(x+2)$   
 $x = -2$

$m=0$

at  $x=-2$ , the slope of  $f$  is 0

$x$	$f'(x)$
-4	12
-3	3
-2	0
-1	3
0	12

$$\begin{aligned}f''(x) &= 3(2x + 4) \\&= 6x + 12 \\&= 6(x+2)\end{aligned}$$

$x$	$f''(x)$
-4	-12
-3	-6
-2	0
-1	6
0	12

$f'''(x) = 6$

$x$	$f'''(x)$
	6

c)  $f(x) = 3x - 7$        $m = 4$   
 $f'(x) = 3$   
 $4 \neq 3$

There are no  $x$ -values  
in which the slope is 4

d) Write the equation of a tangent line  
when  $m=4$  for

$$f(x) = 3x^2 - 2x + 7$$

$$f'(x) = 6x - 2$$

$$4 = 6x - 2$$

$$x = 1$$

$$y = f'(a)(x - a) + f(a)$$

$$y = 4(x - 1) + 8$$

$$\boxed{y = 4x + 4}$$

$$m = 4$$

$$(1, 8)$$

$$f(1) = 3(1)^2 - 2(1) + 7 \\ = 8$$

e) When is the derivative undefined?

$$f(x) = \frac{3x+1}{x-2}$$

$$\begin{matrix} 3x+1 & \curvearrowleft & x-2 \\ 3 & & 1 \end{matrix}$$

$$f'(x) = \frac{(3)(x-2) - (1)(3x+1)}{(x-2)^2}$$

$$f'(x) = \frac{3x-6 - 3x-1}{(x-2)^2} = \frac{-7}{(x-2)^2}$$

Undef  $\Rightarrow$  denom = 0

$$(x-2)^2 = 0$$

$$x = 2$$

The derivative is undefined when  $x = 2$ .