10 Derivatives of Inverse Functions
#1-14

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date per

1. $\frac{d}{dx}[\sin^{-1}(x)]$

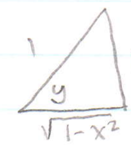
let $y=\sin^{-1}(x)$

$\sin(y)=x$

$\frac{d}{dx}[\sin y]=\frac{d}{dx}[x]$

$\cos y\cdot\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{1}{\cos y}=\boxed{\frac{1}{\sqrt{1-x^2}}}$



$1^2=x^2+a^2$

$a^2=1-x^2$

$\cos y=\frac{\sqrt{1-x^2}}{1}$

2. $\frac{d}{dx}[\cos^{-1}(x)]$

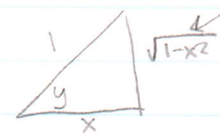
let $y=\cos^{-1}(x)$

$\cos y=x$

$\frac{d}{dx}[\cos y]=\frac{d}{dx}[x]$

$-\sin y\cdot\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{-1}{\sin y}=\boxed{\frac{-1}{\sqrt{1-x^2}}}$



$\sin y=\frac{\sqrt{1-x^2}}{1}$

3. $\frac{d}{dx}[\tan^{-1}(x)]$

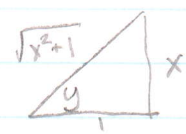
let $y=\tan^{-1}(x)$

$\tan(y)=x$

$\frac{d}{dx}[\tan y]=\frac{d}{dx}[x]$

$\sec^2 y\cdot\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{1}{\sec^2 y}=\frac{1}{(\sqrt{x^2+1})^2}=\boxed{\frac{1}{x^2+1}}$



$1^2+x^2=c^2$

$\sec y=\frac{\sqrt{x^2+1}}{1}$

4. $\frac{d}{dx}[\csc^{-1}(x)]$

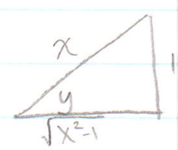
let $y=\csc^{-1}(x)$

$\csc y=x$

$\frac{d}{dx}[\csc y]=\frac{d}{dx}[x]$

$-\csc y\cdot\cot y\cdot\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{-1}{\csc y\cot y}=\boxed{\frac{-1}{x\sqrt{x^2-1}}}$



$1^2+a^2=x^2$

$a^2=x^2-1$

$\csc y=\frac{x}{1}$ $\cot y=\frac{\sqrt{x^2-1}}{1}$

5. $\frac{d}{dx}[\sec^{-1}(x)]$

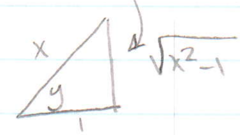
let $y=\sec^{-1}(x)$

$\sec y=x$

$\frac{d}{dx}[\sec y]=\frac{d}{dx}[x]$

$\sec y\cdot\tan y\,\frac{dy}{dx}=1$

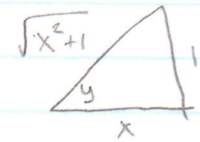
$\frac{dy}{dx}=\frac{1}{\sec y\tan y}=\boxed{\frac{1}{x\sqrt{x^2-1}}}$



$\sec y=\frac{x}{1}$ $\tan y=\frac{\sqrt{x^2-1}}{1}$

6. $\frac{d}{dx}[\cot^{-1}(x)]$

let $y = \cot^{-1}(x)$



$\cot(y) = x$

$\frac{d}{dx}[\cot(y)] = \frac{d}{dx}[x]$

$-\csc^2 y \frac{dy}{dx} = 1$

$\csc y = \frac{\sqrt{x^2+1}}{1}$

$\frac{dy}{dx} = \frac{-1}{\csc^2 x} = \frac{-1}{(\sqrt{x^2+1})^2} = \boxed{\frac{-1}{x^2+1}}$

7. $f(x) = x^3 + 2x - 1$ $\quad a = 2$ $\quad (f^{-1})'(2) = ?$

① on $f^{-1}(2) = ?$

$f(?) = 2 \rightarrow 2 = x^3 + 2x - 1$

let $x=0$ $\quad 2 \neq 0^3 + 20 - 1$

$x=1$ $\quad 2 = 1^3 + 2(1) - 1$ ✓

$f(1) = 2$

② $f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2 = 5$

③ $(f^{-1})'(2) = \frac{1}{f'(1)} = \boxed{\frac{1}{5}}$

8. $f(x) = 2x^5 + x^3 + 1$ $\quad (f^{-1})'(-2) = ?$

① $f^{-1}(-2) = ?$

$f(?) = -2 \rightarrow -2 = 2x^5 + x^3 + 1$

let $x=0$ $\quad -2 \neq 2(0)^5 + 0^3 + 1$

$x=-1$ $\quad -2 = 2(-1)^5 + (-1)^3 + 1$ ✓

$f(-1) = -2$

② $f'(x) = 10x^4 + 3x^2$

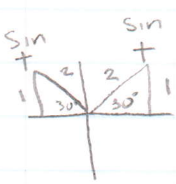
$f'(-1) = 10(-1)^4 + 3(-1)^2 = 13$

③ $(f^{-1})'(-2) = \frac{1}{f'(-1)} = \boxed{\frac{1}{13}}$

9. $f(x) = \sin x$ $\quad -\pi/2 \leq x < \frac{\pi}{2}$ $\quad (f^{-1})'(\frac{1}{2}) = ?$

① $f^{-1}(\frac{1}{2}) = ?$

$f(?) = \frac{1}{2} \rightarrow \frac{1}{2} = \sin x$



$x = 30°, 150°$

$(\pi/6$ or $5\pi/6)$

$f(\pi/6) = \frac{1}{2}$

② $f'(x) = \cos x$

$f'(\pi/6) = \cos \pi/6 = \sqrt{3}/2$

③ $(f^{-1})'(\frac{1}{2}) = \frac{1}{f'(\pi/6)} = \boxed{\frac{2}{\sqrt{3}}}$

10. $f(x) = \cos(2x)$ $\quad (f^{-1})'(1) =$

① $f^{-1}(1) = ?$

$f(?) = 1 \rightarrow 1 = \cos 2x$

$\cos\theta = 1$ $\rightarrow$ $2x = 0$

$\rightarrow \theta = 0$ $\quad x = 0$

$f(0) = 1$

② $f'(x) = -\sin(2x) \cdot 2$

$f'(0) = -2 \cdot \underbrace{\sin(2\cdot 0)}_{0} = 0$

③ $(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{0} = \boxed{\text{undef}}$

11. $f(x) = \sqrt{x-4}$ $\quad (f^{-1})'(6) =$

① $f^{-1}(6) = ?$

$f(?) = 6 \longrightarrow 6 = \sqrt{x-4}$

$36 = x - 4$

$x = 40$

$f(40) = 6$

② $f'(x) = \frac{1}{2}(x-4)^{-1/2} = \frac{1}{2\sqrt{x-4}}$

$f'(40) = \frac{1}{2\sqrt{40-4}} = \frac{1}{2\sqrt{36}} = \frac{1}{12}$

$(f^{-1})'(6) = \frac{1}{f'(40)} = \boxed{12}$

12. $f(x) = x^3 - 7x^2 + 25x - 39$ $\quad g'(0) =$

① $g(0) = ?$

$f(?) = 0 \rightarrow 0 = x^3 - 7x + 25 - 39$

	1	-7	25	-39
1	1	-6	19	-20
-1	1	-8	33	-72
→ 3	1	-4	13	0

$f(3) = 0$

② $f'(x) = 3x^2 - 14x + 25$

$f'(3) = 3(3)^2 - 14(3) + 25$

$= 27 - 42 + 25$

$= 10$

③ $g'(0) = \frac{1}{f'(3)} = \boxed{\frac{1}{10}}$ $\boxed{C}$

13. $f(x) = x^{-1/3}$

$y = \frac{1}{x^{1/3}} \xrightarrow{\text{inv}} x = \frac{1}{y^{1/3}}$

$y^{1/3} \cdot x = 1$

$(y^{1/3})^3 = \left(\frac{1}{x}\right)^3$

$y = \frac{1}{x^3}$

$f^{-1}(x)$

$\frac{d}{dx}[y] = \frac{d}{dx}[x^{-3}]$

$(f^{-1})'(x) = -3x^{-4}$ $\quad \boxed{E}$

14. $f(x) = x^3 - x - 6$

long way

① $g(0) = ?$

$f(?) = 0 \quad 0 = x^3 - x - 6$

	1	0	-1	-6
1	1	1	0	
-1	1	-1	0	
→ 2	1	2	3	0

$f(2) = 0$

$\rightarrow g(0) = 2$

② $f'(x) = 3x^2 - 1$

$f'(2) = 3(2)^2 - 1 = 11$

③ $g'(0) = \frac{1}{11}$

$f'(g(0)) \cdot g'(0)$

$f'(2) \cdot g'(0) = 11 \cdot \frac{1}{11} = \boxed{1}$ $\boxed{B}$

short way

$* \; g'(x) = \frac{1}{f'(g(x))}$

$\rightarrow f'(g(x)) \cdot g'(x) = 1$

$\quad\; 0 \qquad\quad 0 \qquad \boxed{B}$