AP Calculus
11 Related Rates

Name: _____
Date: _____ Per: _____

11 Related Rates

Work on a THIS paper, annotate!!

1) If $y = x^3 + 4x^2$ and $\frac{dx}{dt} = 3$ compute $\frac{dy}{dt}$ when $x = 1$.

$y = x^3 + 4x^2$

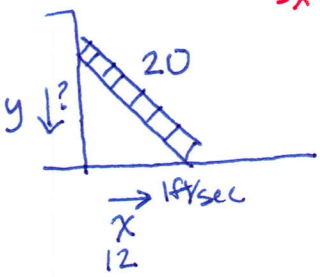
$\frac{dy}{dt} = 3x^2\frac{dx}{dt} + 8x\frac{dx}{dt}$

$\frac{dy}{dt} = 3(1)^2(3) + 8(1)(3)$

$= 9 + 24$

$\frac{dy}{dt} = 33$

2) A ladder 20 feet (constant) long is placed against a wall. The foot of the ladder begins to slide away from the wall at the rate of 1 ft/sec (dx/dt). How fast is the top of the ladder sliding down (dy/dt) the wall when the ladder is 12 feet (x) from the wall?



$x^2 + y^2 = 20^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$(12)(1) + (16)\frac{dy}{dt} = 0$

$16 \cdot \frac{dy}{dt} = -12$

$\frac{dy}{dt} = \frac{-12}{16} = \frac{-3}{4}$ (neg means down)

$y^2 + x^2 = 20^2$

$y^2 + 12^2 = 20^2$

$y = 15$

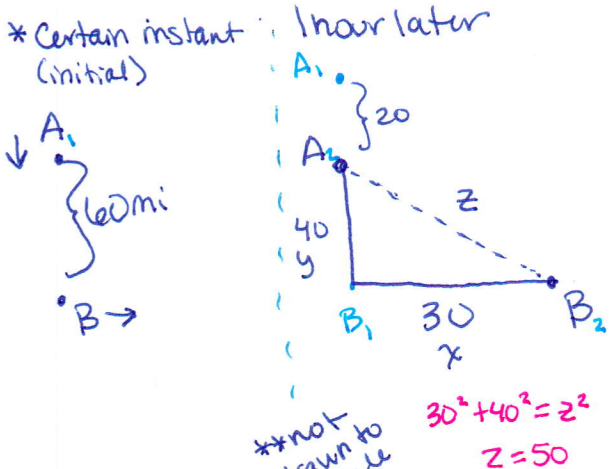
The top of the ladder is sliding down the wall at a rate of $-\frac{3}{4}$ ft/sec when the ladder base is 12 ft from the wall.

3) At a certain instant, car A is 60 miles north of car B. A is traveling south at a rate of 20 mph (dx/dt) while B is traveling east at 30 mph (dy/dt). How fast is the distance between them changing (dz/dt) 1 hour later?

In 1 hr A → 20 mi; 60 − 20 = x

−20 b/c distance getting smaller

in 1 hr B → 30 mi (y)



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(30)(30) + (40)(-20) = (50)\frac{dz}{dt}$

$900 - 800 = 50\frac{dz}{dt}$

$100 = 50\frac{dz}{dt}$

$\frac{dz}{dt} = 2$

$30^2 + 40^2 = z^2$

$z = 50$

The distance between the cars is changing (increasing) at a rate of 2 mi/hr 1 hour after the initial measurement.

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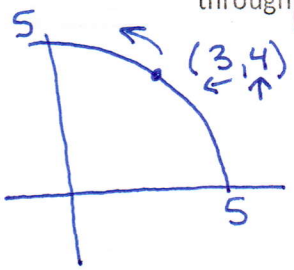
4) A point is moving along the circle $x^2 + y^2 = 25$ in the first quadrant in such a way that its x-coordinate changes at the rate of 2 cm/sec. How fast is its y-coordinate changing as the point passes through $(3, 4)$?



$r = 5$; dx/dt ; dy/dt ; x, y

$x^2 + y^2 = 25$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$3(-2) + (4)\frac{dy}{dt} = 0$

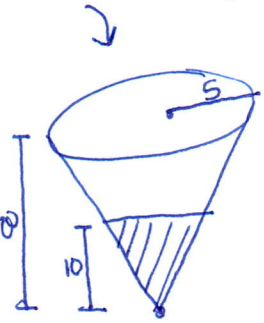
$-6 + 4\frac{dy}{dt} = 6$

$4\frac{dy}{dt} = 6$

$\frac{dy}{dt} = \frac{3}{2}$

The y-coordinate is changing at a rate of $\frac{3}{2}$ cm/sec as the point passes through $(3,4)$.

5) Water is being pumped into a conical tank at the rate of 100 ft^3/min. The height of the tank is 20 ft and its radius is 5 ft. How fast is the water level rising when the water height is 10 ft?



dV/dt ; dh/dt ; h

$\frac{h}{r} = \frac{20}{5}$

$h = 4r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$

$V = \frac{1}{3}\pi\frac{h^2}{16}\cdot h$

$V = \frac{\pi}{48}h^3$

$\frac{dV}{dt} = \frac{3\pi}{48}h^2\frac{dh}{dt}$

$100 = \frac{\pi}{16}(10)^2\cdot\frac{dh}{dt}$

$\frac{16}{\pi} = \frac{dh}{dt}$

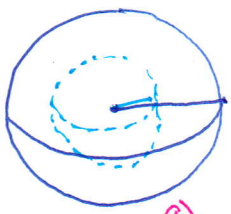
≈ 5.09296

The water level is rising at a rate of 5.09296 ft/min when the water height is 10 ft.

6) A spherical snowball is melting in such a way that its surface area decreases at the rate of 1 in^2/min.

a. How fast is the radius shrinking when $r = 3$?

b. How fast is the radius shrinking when $r = 8$?



ds/dt ; $-$ b/c shrinking ; dr/dt ; dr/dt

$S = 4\pi r^2$

$\frac{ds}{dt} = 8\pi r\frac{dr}{dt}$

a) $-1 = 8\pi(3)\frac{dr}{dt}$

$\frac{-1}{24\pi} = \frac{dr}{dt} \approx -.013263$

b) $-1 = 8\pi(8)\frac{dr}{dt}$

$\frac{-1}{64\pi} = \frac{dr}{dt} \approx -.00497$

The radius is shrinking at a rate of

a) $-.013$ in/min when $r = 3$.

b) $-.004$ in/min when $r = 8$.

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