12 Linear Approximation
#1-13

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1. $f(x) = x^3 - 2x + 3$ $\quad x = 2$

$f(2) = (2)^3 - 2(2) + 3 = 7$ $\quad (2, 7)$ $m = 10$

$f'(x) = 3x^2 - 2$

$f'(2) = 3(2)^2 - 2 = 10$ $\quad$ tan line $\quad y = 10(x-2) + 7$

$y(2.01) = 10(2.01 - 2) + 7$

$= 7.1$

$\boxed{f(2.01) \approx 7.1}$

2. $f(x) = \sqrt{x^2 + 9}$ $\quad x = -4$

$f(-4) = \sqrt{(-4)^2 + 9} = 5$ $\quad (-4, 5)$ $m = -4/5$

$f'(x) = \dfrac{2x}{2\sqrt{x^2+9}}$

$f'(-4) = \dfrac{-4}{\sqrt{(-4)^2+9}} = -4/5$ $\quad$ tan line $\quad y = -4/5(x+4) + 5$

$y(-3.99) = -4/5(-3.99 + 4) + 5$

$= 4.992$

$\boxed{f(-3.99) \approx 4.992}$

3. $f(x) = x + \frac{1}{x}$ $\quad x = 1$

$f(1) = 1 + \frac{1}{1}$ $\quad (1, 2)$ $m = 0$

$f'(x) = 1 - \frac{1}{x^2}$

$f'(1) = 1 - \frac{1}{1^2} = 0$ $\quad$ tan line $\quad y = 0(x-1) + 2$

$y(1.01) = 0(1.01 - 1) + 2$

$= 2$

$\boxed{f(1.01) \approx 2}$

4. $f(x) = \tan x$ $\quad x = \pi$

$f(\pi) = \tan \pi = 0$ $\quad (\pi, 0)$ $m = 1$

$f'(x) = \sec^2 x$

$f'(\pi) = (\sec \pi)^2 = (-1)^2 = 1$ $\quad$ tan line $\quad y = 1(x - \pi) + 0$

$y(\pi + .01) = 1(\pi + .01 - \pi)$

$= .01$

$\boxed{f(\pi + .01) \approx .01}$

5. Let $f(x)=\sqrt{x}$ $\quad x=100$

$f(100)=\sqrt{100}=10$ $\quad (100,10)$ $m=1/20$

$f'(x)=\dfrac{1}{2\sqrt{x}}$

$f'(100)=\dfrac{1}{2\sqrt{100}}=\dfrac{1}{20}$

tan line $y=\frac{1}{20}(x-100)+10$

$y(101)=\frac{1}{20}(101-100)+10$

$=\frac{1}{20}+10$

$f(101)=\boxed{\sqrt{101}\approx 10.05}$

6. Let $f(x)=\sqrt[3]{x}$ $\quad x=27$

$f(27)=\sqrt[3]{27}=3$ $\quad (27,3)$ $m=\frac{1}{27}$

$f'(x)=\dfrac{1}{3\sqrt[3]{x^2}}$

$f'(27)=\dfrac{1}{3\sqrt[3]{27^2}}=\dfrac{1}{27}$

tan line $y=\frac{1}{27}(x-27)+3$

$y(26)=\frac{1}{27}(26-27)+3$

$=2.9629$

$f(26)=\boxed{\sqrt[3]{26}\approx 2.9629}$

7. $(3,1/3)$ $\quad m=-1/9$ $\quad$ tan line $y=-\frac{1}{9}(x-3)+\frac{1}{3}$

$y(3.4)=-\frac{1}{9}(3.4-3)+\frac{1}{3}$

$=.288$

$\boxed{f(3.4)\approx 0.2889}$

8. Let $f(x)=\sqrt{x}$ $\quad x=144$

$f(144)=\sqrt{144}=12$ $\quad (144,12)$ $m=1/24$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f'(144)=\frac{1}{2\sqrt{144}}=\frac{1}{24}$

tan line $y=\frac{1}{24}(x-144)+12$

$y(139)=\frac{1}{24}(139-144)+12$

$=11.583$

$f(139)=\boxed{\sqrt{139}\approx 11.583}$

must work in radians

9. Let $f(x) = \sin x$ $\quad x = 120°$ $\quad (2\pi/3, \sqrt{3}/2)$ $\quad m = -1/2$

$f(120°) = \sin(120°) = \sqrt{3}/2$

$f'(x) = \cos x$

$f'(120°) = \cos(120°) = -1/2$

tan line $y = -\frac{1}{2}\left(x - \frac{2\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$y(122) = -\frac{1}{2}\left(\frac{61\pi}{90} - \frac{60\pi}{90}\right) + \frac{\sqrt{3}}{2}$

$= .8485$

$122 \cdot \frac{\pi}{180} = \frac{61\pi}{90}$

$f(122°) = \sin(122°) \approx 0.8485$

10. Let $f(x) = \sqrt[4]{x}$ $\quad x = 81 = 3^4$ $\quad (81, 3)$ $\quad m = \frac{1}{108}$

$f(81) = \sqrt[4]{81} = 3$

$f'(x) = \frac{1}{4\sqrt[4]{x^3}}$

$f'(81) = \frac{1}{4\sqrt[4]{81}^3} = \frac{1}{108}$

tan line $y = \frac{1}{108}(x - 81) + 3$

$y(78) = \frac{1}{108}(78 - 81) + 3$

$= -3/108 + 3$

$\approx -.027 + 3 \rightarrow$ **2.97 E**

$108 \overline{)3.00}$ $\quad .027$

-216

840

756

11. $(5, 10)$ $\quad m = 2$ $\quad$ tan line $y = 2(x - 5) + 10$

$y(5.5) = 2(5.5 - 5) + 10$

$= 2(.5) + 10 = 1 + 10 =$ **11 C**

12. $f(x) = \cos x$ $\quad x = \pi/3$

$f(\pi/3) = \cos(\pi/3) = 1/2$ $\quad (\pi/3, 1/2)$ $\quad m = -\sqrt{3}/2$

$f'(x) = -\sin x$

$f'(\pi/3) = -\sin \pi/3 = -\sqrt{3}/2$

tan line $y = -\frac{\sqrt{3}}{2}\left(x - \pi/3\right) + \frac{1}{2}$

$y(62°) = -\frac{\sqrt{3}}{2}\left(\frac{31\pi}{90} - \frac{30\pi}{90}\right) + 1/2$

≈ 0.46977

$62° \cdot \frac{\pi}{180} = \frac{31\pi}{90}$

$f(62°) = \cos(62°) \approx$ **.46977 B**

13. * typo Should be $(5.2)^3$ K

$(5.2)^2$ → let $f(x) = x^2$ $x = 5$

$f(5) = 25$ $(5, 25)$ $m = 10$

$f'(x) = 2x$

$f'(5) = 10$ tan line $y = 10(x-5) + 25$

$y(5.2) = 10(5.2-5) + 25$

$= \boxed{27}$

$(5.2)^3$ → let $f(x) = x^3$ $x = 5$

$f(5) = 125$ $(5, 125)$ $m = 75$

$f'(x) = 3x^2$

$f'(5) = 3(5)^2 = 75$ tan line $y = 75(x-5) + 125$

$y(5.2) = 75(5.2-5) + 125$

$= 15 + 125 = \boxed{140}$

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