Mean Value Theorem
1 - 9

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1. f(x) is cts + differentiable on [-2, 1] so we CAN apply MVT

$f'(x) = 2x$ $\quad f(1) = 1^2 = 1$ $\quad f(-2) = (-2)^2 = 4$

$f'(c) = 2c$

$f'(c) = \dfrac{f(1) - f(-2)}{1 - -2}$

$2c = \dfrac{1 - 4}{3} = \dfrac{-3}{3}$

$2c = -1$

$c = -1/2$

2. f(x) is cts + differentiable on [-1, 1] so we CAN apply MVT

$f'(x) = 3x^2 + 2$ $\quad f(1) = (1)^3 + 2(1) = 3$ $\quad f(-1) = (-1)^3 + 2(-1) = -3$

$f'(c) = 3c^2 + 2$

$f'(c) = \dfrac{f(1) - f(-1)}{1 - -1}$

$3c^2 + 2 = \dfrac{3 - -3}{1 - -1} = \dfrac{6}{2}$

$3c^2 + 2 = 3$

$3c^2 = 1$

$c^2 = 1/3$

$c = \pm\sqrt{1/3}$

3. f(x) is not cts on [-1, 2] at x=0 so we CAN NOT use MVT

4. f(x) is cts + diffable on [1, 4] so we can use MVT

$\frac{x+1}{1} \times \frac{x}{1}$

$f'(x) = \dfrac{x - (x+1)}{(x)^2} = \dfrac{-1}{x^2}$ $\quad f(4) = \dfrac{4+1}{4} = \dfrac{5}{4}$ $\quad f(1) = \dfrac{1+1}{1} = 2$

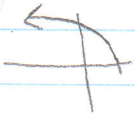
$f'(c) = -1/c^2$

$-\dfrac{1}{c^2} = \dfrac{\frac{5}{4} - \frac{8}{4}}{4 - 1} = \dfrac{-3}{4} \div 3 = -\dfrac{3}{4} \cdot \dfrac{1}{3}$

$-\dfrac{1}{c^2} = -\dfrac{1}{4}$

$c^2 = 4$

$c = \pm 2$



5. $f(x)$ is cts + diffable on $[-7, 2]$ so we can use MVT

$f'(x) = \frac{1}{2}(2-x)^{-1/2}(-1)$ $\quad f(2) = \sqrt{2-2} = 0$ $\quad f(-7) = \sqrt{2 - -7} = 3$

$f'(c) = \frac{-1}{2\sqrt{2-c}}$

$$\frac{-1}{2\sqrt{2-c}} = \frac{0-3}{2 - -7} = \frac{-3}{9} = \frac{-1}{3}$$

$$2\sqrt{2-c} = 3$$
$$(\sqrt{2-c})^2 = \left(\frac{3}{2}\right)^2$$
$$2 - c = \frac{9}{4}$$
$$\frac{8}{4} - \frac{9}{4} = c = -\frac{1}{4}$$

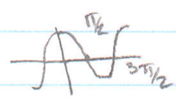
6. $f(x)$ is cts + diffable on $[-\pi, \pi]$ so we can use MVT

$f'(x) = 1 - 2\cos x$ $\quad f(\pi) = \pi - 2\sin(\pi) = \pi$ $\quad f(-\pi) = -\pi - 2\sin(-\pi) = \pi$

$f'(c) = 1 - 2\cos c$

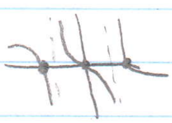
$$1 - 2\cos c = \frac{\pi - -\pi}{\pi - -\pi} \quad \frac{2\pi}{2\pi} = 1$$

$$1 - 2\cos c = 1$$
$$-2\cos c = 0$$
$$\cos c = 0$$
$$c = \cos^{-1}(0) = \pi/2, -\pi/2$$



7. False $\quad x \neq 0$ for $f(x) = \frac{1}{x}$ so f is not cts on the interval

8. False



$\leftarrow$ 3 x-int but no horizontal tangents

9. True Because a polynomial fn is continuous so the MVT applies