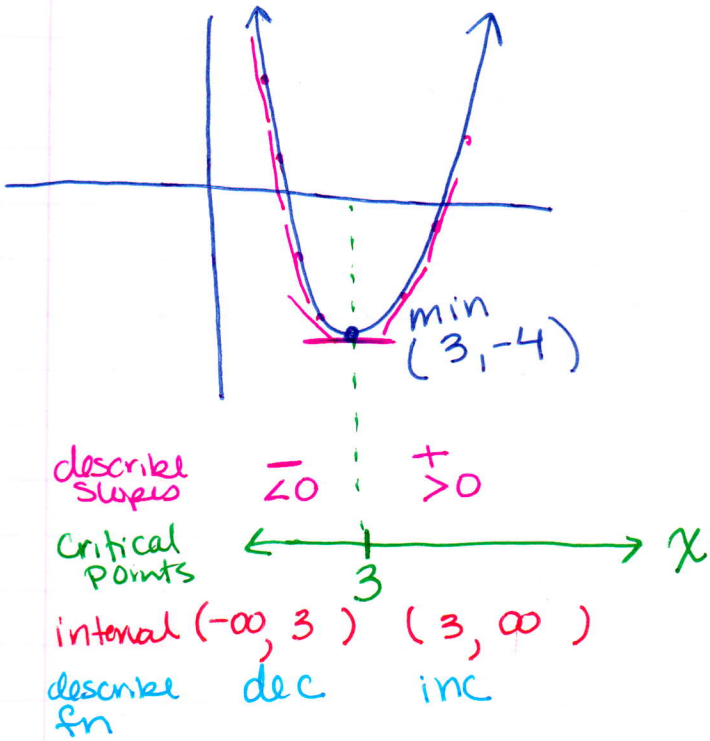
Pre 15 notes

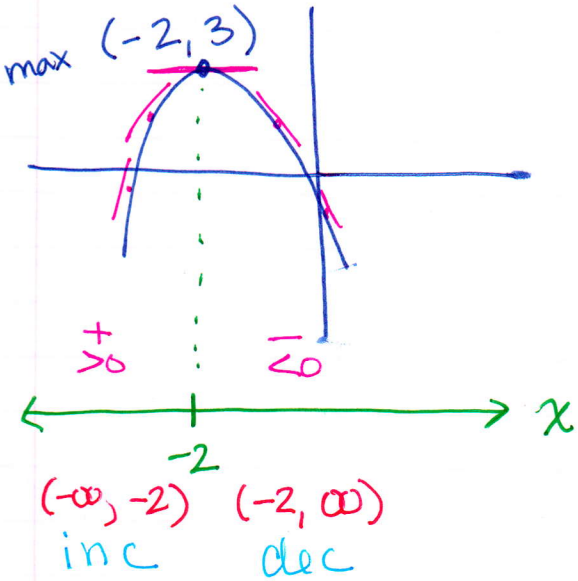
No title

$f(x) = 2x^2 - 12x + 14$



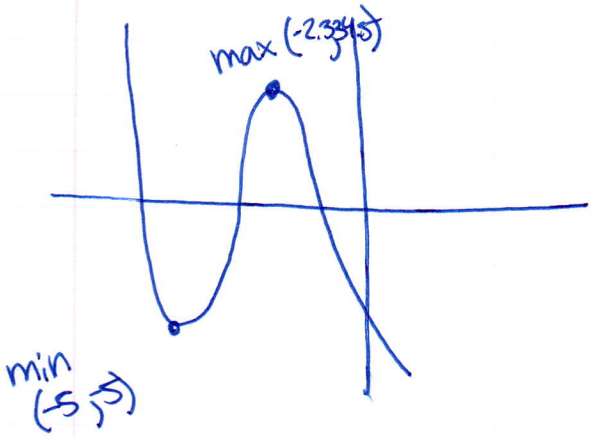
- Since $f'(x) < 0$ on the interval $(-\infty, 3)$ the function is dec.
- Since $f'(x) > 0$ on the interval $(3, \infty)$ the function is inc.

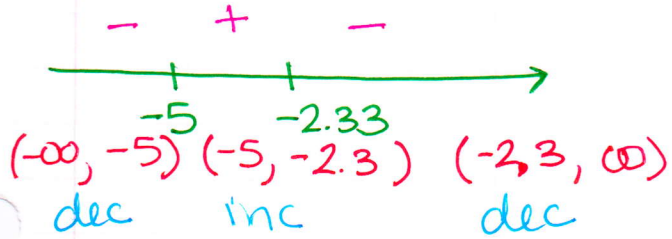
$f(x) = -x^2 - 4x - 1$



- Since $f'(x) > 0$ on the interval $(-\infty, -2)$ the function is increasing.
- Since $f'(x) < 0$ on the interval $(-2, \infty)$ the function is decreasing.

$f(x) = -x^3 - 11x^2 - 35x - 30$





- Since $f'(x) < 0$ on the intervals $(-\infty, -5)$ and $(-2.3, \infty)$ the function is decreasing.

- Since $f'(x) > 0$ on the interval $(-5, -2.3)$ the function is increasing.

15. First Derivative Test

Recall "No title" notes

$f'(x) > 0$ then the fn is inc
$f'(x) < 0$ fn is dec
$f'(x) = 0$ or DNE } Critical point

sign chart

Ex 1

$f(x) = 2x^2 - 12x + 14$
$f'(x) = 4x - 12$
$0 = 4x - 12$
$x = 3$

f'	$-$	$+$
CP	3	
int	$(-\infty, 3)$	$(3, \infty)$
f	dec	inc

$f'(0) = 4(0) - 12 < 0$
$f'(10) = 4(10) - 12 > 0$

Ex 2

$f(x) = \frac{1}{x} = x^{-1}$
$f'(x) = -x^{-2} = -1/x^2$
$x = 0$ is a CP

f'	$-$	$-$
CP	0	
int	$(-\infty, 0)$	$(0, \infty)$
f	dec	dec

$f'(-10) = \frac{-1}{(-10)^2} < 0$

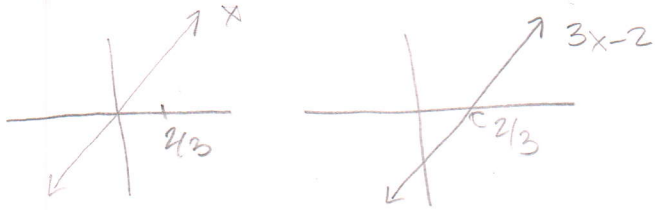
$f'(10) = \frac{-1}{10^2} < 0$

Ex 3

$f(x) = x^3$
$f'(x) = 3x^2$
$0 = 3x^2$
$x = 0$

f'	$+$	$+$
CP	0	
int	$(-\infty, 0)$	$(0, \infty)$
f	inc	inc

CP
$(-\infty, 0)$ $f'(\ ) = 3(\ )^2 > 0$
$(0, \infty)$ $f'(\ ) = 3(\ )^2 > 0$



Ex4 $f(x) = x^3 - x^2$

$f'(x) = 3x^2 - 2x$

$0 = x(3x-2)$

CP $x = 0, 2/3$

		max @ x=0		min @ x=2/3	
f'	+		−		+
CP		0		2/3	
int	$(-\infty, 0)$		$(0, 2/3)$		$(2/3, \infty)$
f	inc		dec		inc

$(-\infty, 0)$ $f'(\ ) = (-)(-) > 0$

$(0, 2/3)$ $f'(\ ) = (+)(-) < 0$

$(2/3, \infty)$ $f'(\ ) = (+)(+) > 0$

Ex5 $f(x) = \frac{x^3}{3} - \frac{x^2}{3} - \frac{x}{3}$

$\frac{1}{3}(3x^2 - 2x - 1) = f'(x) = x^2 - \frac{2}{3}x - \frac{1}{3}$

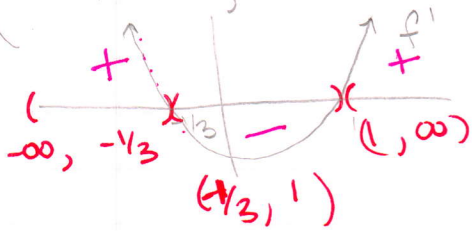
$\left(0 = x^2 - \frac{2}{3}x - \frac{1}{3}\right)3$

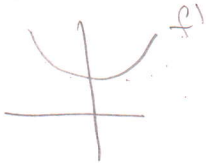
$0 = 3x^2 - 2x - 1$

$0 = (3x+1)(x-1)$

CP $x = -1/3, 1$

f'	+		−		+
CP		$-1/3$		1	
int	$(-\infty, -1/3)$		$(-1/3, 1)$		$(1, \infty)$
f	inc		dec		inc





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Ex6 $f(x) = \dfrac{x}{x-2}$ Domain: $x \neq 2$ VA

$x \quad x-2$

$$f'(x) = \frac{(x-2) - x}{(x-2)^2}$$

$$f'(x) = \frac{-2}{(x-2)^2}$$

f'	$-$		$-$
CP		2	
int	$(-\infty, 2)$		$(2, \infty)$
f	dec		dec

$$f'(x) = \frac{-2}{(x-2)^2} \rightarrow \frac{(-)}{(+)} < 0$$

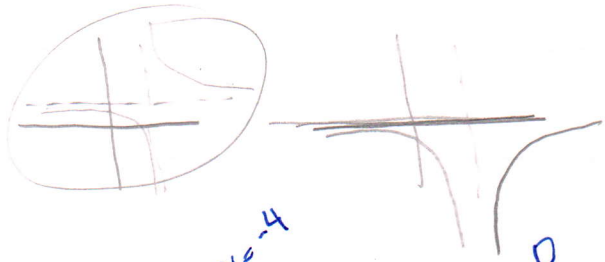
$f'(x) = 0$

$-2 \neq 0$

$f'(x)$ DNE

$(x-2)^2 = 0$

$x = 2$



Ex7 $f(x) = \dfrac{x^2}{3x+6}$ Domain $x \neq -2$ VA

$x^2 \quad 3x+6$

$2x \quad 3$

$$f'(x) = \frac{2x(3x+6) - 3x^2}{(3x+6)^2}$$

$$f'(x) = \frac{6x^2 + 12x - 3x^2}{(3x+6)^2}$$

f'	$+$	max at $x=-4$	$-$	$-$	min @ $x=0$	$+$
CP		-4		-2	0	
int	$(-\infty, -4)$		$(-4, -2)$		$(-2, 0)$	$(0, \infty)$
f	inc		dec		dec	inc

$$f'(x) = \frac{3x^2 + 12x}{(3x+6)^2} = \frac{3x(x+4)}{(3x+6)^2}$$

$f'(x) = 0$

$3x^2 + 12x = 0$

$3x(x+4) = 0$

$x = 0, -4$

$f'(x)$ DNE

$(3x+6)^2 = 0$

$3x + 6 = 0$

$x = -2$

Ex 8) $f(x) = \dfrac{2x^2}{x-3}$

Domain
$x \neq 3$
VA

$f'(x) = \dfrac{2x^2 - 12x}{(x-3)^2}$

$f'(x) = 0$

$2x^2 - 12x = 0$

$2x(x-6) = 0$

$x = 0, 6$

$f'(x)$ DNE

$(x-3)^2 = 0$

$x = 3$

$(-)(-)$ $(+)(-)$ $(+)(-)$ $(+)(+)$

0 3 6

f'	+ max @ x=0	−	−	+ min @ x=6
CP	0	3	6	
int	$(-\infty, 0)$	$(0, 3)$	$(3, 6)$	$(6, \infty)$
f	inc	dec	dec	inc