Concavity
1-16

Stew Dent
date Per

1. $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

$0 = 6x$

POI $x = 0$

f''	−	+
POI		0
f	CD	CU
	$(-\infty, 0)$	$(0, \infty)$

2. $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$0 = 6x - 12$

POI $x = 2$

f''	−	+
POI		2
f	CD	CU
	$(-\infty, 2)$	$(2, \infty)$

3. $f(x) = 2x^4 - 8x + 3$

$f'(x) = 8x^3 - 8$

$f''(x) = 24x^2$

$0 = 24x^2$

POI $x = 0$

f''	+	+
POI		0
f	CU	CU
	$(-\infty, 0)$	$(0, \infty)$

4. $f(x) = x\sqrt{x+3}$ $\quad x \geq -3$

x	$\sqrt{x+3}$
1	$\frac{1}{2}(x+3)^{-1/2}$

$f'(x) = \sqrt{x+3} + \frac{x}{2\sqrt{x+3}}$

$= \frac{2(x+3)}{2\sqrt{x+3}} + \frac{x}{2\sqrt{x+3}}$

$= \frac{3x+6}{2\sqrt{x+3}}$

$3x+6$	$2\sqrt{x+3}$
3	$\frac{1}{\sqrt{x+3}}$

$f''(x) = \left(\frac{6\sqrt{x+3} - \frac{3x+6}{\sqrt{x+3}}}{(2\sqrt{x+3})^2}\right)\frac{\sqrt{x+3}}{\sqrt{x+3}}$

$f''(x) = \frac{6(x+3) - (3x+6)}{4(x+3)^{2/2}\sqrt{x+3}}$

$f''(x) = \frac{3x+12}{4(x+3)^{3/2}}$

POI $x = -4$ or $x = -3$

↑ not in Domain

f''	+
POI	−3
f	CU
	$(-3, \infty)$

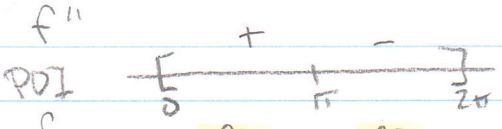
5. $f(x) = x - \sin x$ $\quad [0, 2\pi]$

$f'(x) = 1 - \cos x$

$f''(x) = \sin x$

$0 = \sin x$

POI $x = 0, \pi, 2\pi$



POI		
f	CU	CD
	$(0, \pi)$	$(\pi, 2\pi)$

6. $f(x) = 6x - x^2$ $\quad f''(x) = -2$

$f'(x) = 6 - 2x$ $\quad f''(3) < 0$ CD ∩ $\quad x = 3$ is a max

$0 = 6 - 2x$

CP $x = 3$

7. $f(x) = 5 + 3x^2 - x^3$ $\quad f''(x) = 6 - 6x$

$f'(x) = 6x - 3x^2$ $\quad f''(0) > 0$ CU ∪ $x = 0$ is a min

$0 = 3x(2 - x)$ $\quad f''(2) < 0$ CD ∩ $x = 2$ is a max

CP $x = 0, 2$

8. $f(x) = x^4 - 4x^3 + 2$ $\quad f''(x) = 12x^2 - 24x$

$f'(x) = 4x^3 - 12x^2$ $\quad f''(0) = 0$ no info

$0 = 4x^2(x - 3)$ $\quad f(3) > 0$ CU ∪ $x = 3$ is a min

CP $x = 0, 3$

9. $f(x) = x + \frac{4}{x}$ $\quad f''(x) = \frac{8}{x^3}$

$f'(x) = 1 - \frac{4}{x^2}$ $\quad f''(-2) < 0$ CD ∩ $x = -2$ is a max

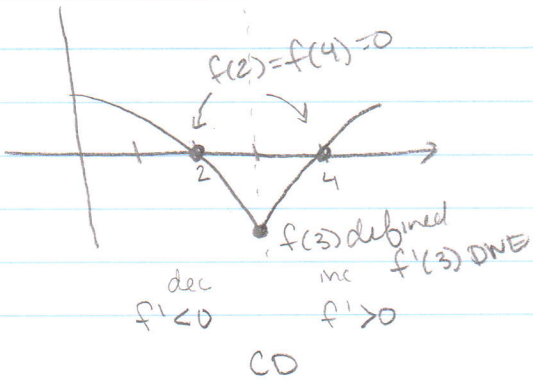
$0 = 1 - \frac{4}{x^2}$ $\quad f''(2) > 0$ CU ∪ $x = 2$ is a min

$0 = x^2 - 4$

$x \neq 0$ VA

CP $x = \pm 2, 0$

10.



11. $y = 2(4-x)^{-1}$

$y' = -2(4-x)^{-2}(-1) = 2(4-x)^{-2}$

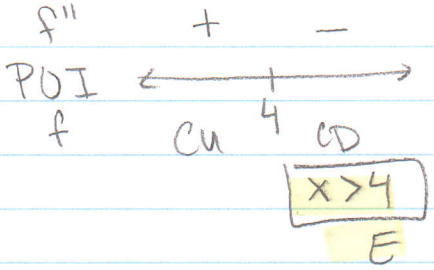
$y'' = 2(-2)(4-x)^{-3}(-1)$

$y'' = \frac{4}{(4-x)^3}$

$y'' \neq 0$ $\quad$ y'' DNE

$(4-x)^3 = 0$

$x = 4$



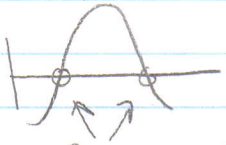
$x > 4$

E

12. $f(x) = x^2 + 5\cos x$ on $[0, 2\pi]$

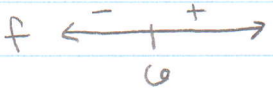
$f'(x) = 2x - 5\sin x$

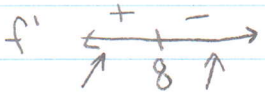
$f''(x) = 2 - 5\cos x$

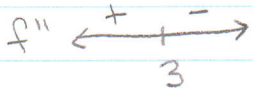


2 changes in concavity for one $(0, 2\pi)$ fn repeats n times $\Rightarrow$ 2n E

13.







C) $f'(4) > 0$

B) $f'(10) < 0$

D) $f'(1) > 0$

E) $f'(-7) > 0$

A) $f''(3) = 0$

14. $f' > 0$ means f inc $\rightarrow$ by table eliminate D, E

thus $f'' > 0$ means f' inc $\rightarrow$ $.998 < x < 1$ f' inc $\rightarrow$ f CU

also f CU

$1 < x < 1.002$ f' dec $\rightarrow$ f CD

} change in concavity

$\Rightarrow$ C

15. $y = 6x^2 + \frac{x}{2} + 3 + 6x^{-1}$

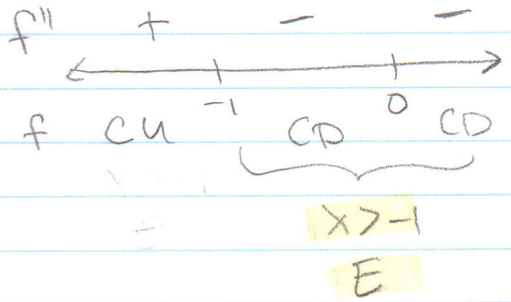
$y' = 12x + \frac{1}{2} - 6x^{-2}$

$y'' = 12 + 12x^{-3}$

$0 = 12 + \frac{12}{x^3}$

$x \neq 0 \quad 0 = 12x^3 + 12$

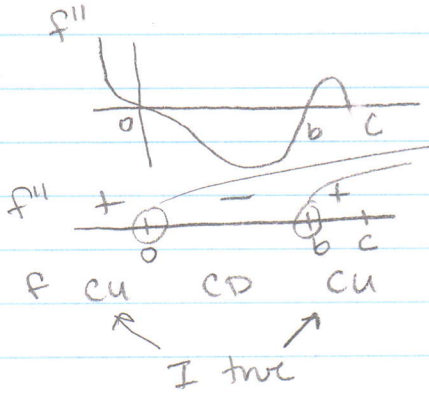
$x = -1$



$x > -1$

E

16.



III is true

not enough info for II

→ I + III D