

16e. Concavity

Concavity is the curvature of a graph.



Concave up

- tan lines lie below the graph
 - tan line approx is an Under estimate
- $$f''(x) > 0$$



Concave down

- tan lines lie above the graph
 - tan line approx is an Over estimate
- $$f''(x) < 0$$

Inc/Dec $\rightarrow f' = 0$

CU/CD $\rightarrow f'' = 0$

$\rightarrow x=a$ critical points

$\rightarrow x=a$ poss points of inflection

Ex 1 $y = x^2 + 6x + 10$

$$y' = 2x + 6$$

$$x = -3$$

f''	+		
f'	-	+	
205 CP			
f_n	dec	-3 min	inc

CU

$$y'' = 2 > 0$$

Ex 2 $y = -x^3 + 4x^2 - 2$

$$y' = -3x^2 + 8x = x(-3x + 8)$$

$$x = 0, \frac{8}{3}$$



f''	+		-	
f'	-	+	-	
205 CP				
f_n	dec	0 min	inc	$\frac{8}{3}$ max dec

CU CD

$$y'' = -6x + 8$$

$$x = \frac{4}{3}$$

$$\underline{\text{Ex 3}} \quad y = x^3 - x^2 + 1$$

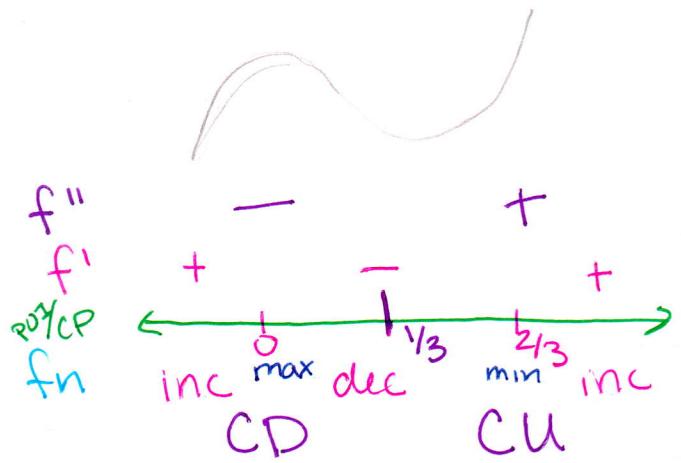
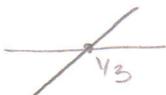
$$y' = 3x^2 - 2x$$

$$x = 0, \frac{2}{3}$$



$$y'' = 6x - 2$$

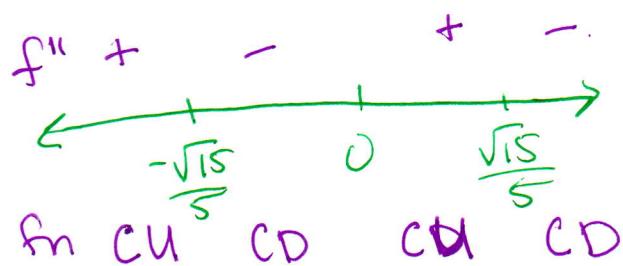
$$x = \frac{1}{3}$$



$$\underline{\text{Ex 4}} \quad y = -x^5 + 2x^3 - 4$$

$$y' = 12x - 20x^3$$

$$x = -\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}$$



Second Derivative Test

Let $x=a$ be a critical point ($f'(a)=0$)

Then $f''(a) > 0$ *f is CU \cup $x=a$ is a min
 $f''(a) < 0$ *f is CD \cap $x=a$ is a max
 $f''(a) = 0$ not enough info

Ex 5) Find the extrema using the 2nd deriv. test *C.P.

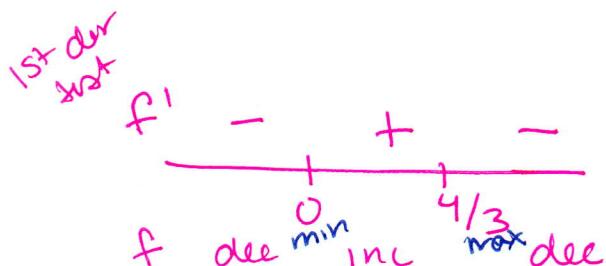
$$y = -x^3 + 2x^2 - 3$$

$$y' = -3x^2 + 4x$$

$$0 = -x(3x-4)$$

$$\text{CP} \quad x = 0, \frac{4}{3}$$

$$y'' = -6x + 4$$



$$y''(0) = -6(0) + 4 > 0 \quad V \quad x=0 \text{ is a min}$$

$$y''(\frac{4}{3}) = -6(\frac{4}{3}) + 4 < 0 \quad \wedge \quad x=\frac{4}{3} \text{ is a max}$$

$$\text{Ex6) } y = x^5 - 2x^3 - 1$$

CP

$$y' = 5x^4 - 6x^2$$

$$0 = x^2(5x^2 - 6)$$

$$x = 0, \pm\sqrt{\frac{6}{5}}$$

$$5x^2 = 6$$

$$x^2 = \frac{6}{5}$$

$$x = \pm\sqrt{\frac{6}{5}}$$

$$y'' = 20x^3 - 12x$$

$$y''(\sqrt{\frac{6}{5}}) = 20(\sqrt{\frac{6}{5}})^3 - 12(\sqrt{\frac{6}{5}}) > 0$$

$$y''(0) = 20(0)^3 - 12(0) = 0$$

$$y''(-\sqrt{\frac{6}{5}}) = 20(-\sqrt{\frac{6}{5}})^3 - 12(-\sqrt{\frac{6}{5}}) < 0$$

$x = \sqrt{\frac{6}{5}}$ is a min
not enough info
 $x = -\sqrt{\frac{6}{5}}$ is a max

$$\text{Ex7) } y = \frac{3}{x^2-1} = 3(x^2-1)^{-1}$$

$x \neq 1, -1$ V.A.

CP

$$y' = 3(-1)(x^2-1)^{-2} \cdot (2x)$$

$$0 = \frac{-6x}{(x^2-1)^2}$$

$$x = 0, +1, -1$$

not points

$$\frac{-6x}{(x^2-1)^2} \gtrless \frac{(x^2-1)^2}{2(x^2-1)(2x)}$$

$$y''(0) = \frac{-6(-1)^2}{((-1)^2)^2} = \frac{-6}{1} < 0 \quad \wedge \quad x=0 \text{ is a max}$$

$$\text{Ex8) } y = x^3 - 4x^2 + 2$$

