Optimization
#1-8

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date Per

1. 

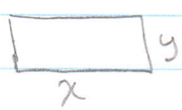
PE. $A = x \cdot y$

SE $100 = 2x + 2y$
$\to y = 50 - x$

optimize
$A = x(50-x)$
$A' = 50 - 2x$
$x = 25 \to y = 50 - 25 = 25$

A' + −
0 25 50
↳max

max area = $(25)(25) = 625\ ft^2$

2. 

PE $P = 2x + 2y$

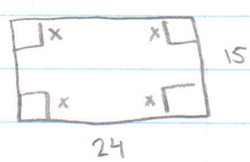
SE $100 = x \cdot y$
$\to y = \frac{100}{x}$

$\frac{d}{dx}[200x^{-1}] = -200x^{-2}$

optimize
$P = 2x + \frac{200}{x}$
$P' = 2 - \frac{200}{x^2}$
$0 = \frac{2x^2 - 200}{x^2}$
$x = 0, \pm 10 \to y = \frac{100}{10} = 10$

P' + −
0 10 100
↳max

max perimeter = $2(10) + 2(10) = 40$ in

3. 

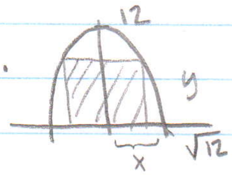
PE $V = x \cdot l \cdot w$

SE $l = 24 - 2x$
$w = 15 - 2x$

optimize
$V = x(24-2x)(15-2x)$
$V = 4x^3 - 78x^2 + 360x$
$V' = 12x^2 - 156x + 360$
$0 = 12(x-10)(x-3)$
$x = 3, \cancel{10} \to l = 24 - 2(3) = 18$
$w = 15 - 2(3) = 9$

V' + −
0 3 7.5

max Volume = $3(18)(9) = 486\ in^3$

4. 

PE $A = 2x \cdot y$ SE $y = 12 - x^2$

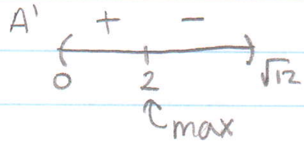
optimize

$A = 2x(12 - x^2) = 24x - 2x^3$

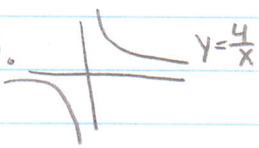
$A' = 24 - 6x^2$

$0 = 6(4 - x^2)$

$x = \pm 2 \rightarrow y = 12 - 2^2 = 8$



max Area $= 2(2)(8) = 32$

5. 

PE $d = \sqrt{(x-0)^2 + (y-0)^2}$ SE $xy = 4$, $y = \frac{4}{x}$

opt

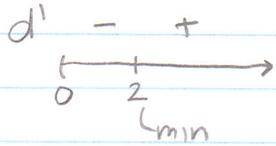
$d = \sqrt{x^2 + \left(\frac{4}{x}\right)^2}$

$d' = \frac{1}{2}\left(x^2 + \frac{16}{x^2}\right)^{-1/2}\left(2x - 32x^{-3}\right)$

$0 = \dfrac{2\left(x - \frac{16}{x^3}\right)}{2\sqrt{x^2 + \frac{16}{x^2}}}$ $\rightarrow$ $=0$ when $x = 0, \pm 2$

$\leftarrow$ always >0

$y = \frac{4}{2} = 2$



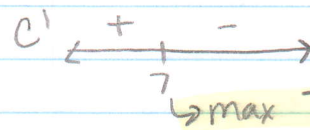
min distance $= \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\boxed{D}$

6. catch limit $= x(15 - x) - x$ want max catch

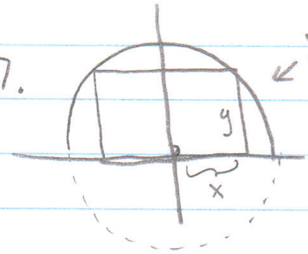
$C = 15x - x^2 - x = 14x - x^2$

$C' = 14 - 2x$

$x = 7$



$\boxed{B}$

7. 

$x^2+y^2=8^2$

PE

$A = 2x \cdot y$

SE

$x^2+y^2=64$

$\rightarrow y=\sqrt{64-x^2}$

$2x \quad (64-x^2)^{1/2}$

$2 \quad \frac{1}{2}(64-x^2)^{-1/2} \cdot (-2x)$

$\hookrightarrow \frac{-x}{\sqrt{64-x^2}}$

optimize

$A = 2x(64-x^2)^{1/2}$

$A' = 2\sqrt{64-x^2} + \frac{-2x^2}{\sqrt{64-x^2}}$

$A' = \frac{2(64-x^2)}{\sqrt{64-x^2}} + \frac{-2x^2}{\sqrt{64-x^2}} = \frac{128-4x^2}{\underbrace{\sqrt{64-x^2}}_{\text{always} > 0 \text{ since } x<8}}$

$x = \pm\sqrt{32} = \pm 4\sqrt{2}$

$y = \sqrt{64-\sqrt{32}^2} = \sqrt{64-32} = \sqrt{32}$

max Area $= 2\sqrt{32}\sqrt{32} = 64$ E

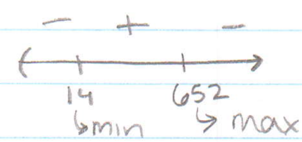
8. Profit = Sold for − manufacturing cost

$P = (75-.01x) - (1850 + 28x - x^2 + .001x^3)$

$= -.001x^3 + x^2 - 28.01x - 1775$

$P' = -.003x^2 + 2x - 28.01$

$x = 14.3123, 652.354$



$x = 652$ $\boxed{B}$