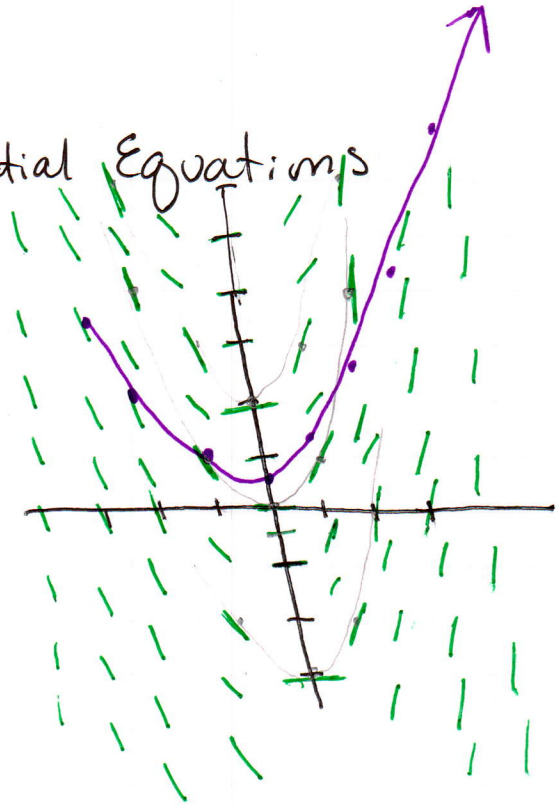
20 Slope fields and Differential Equations

Review



1. Sketch (on the same graph)

$y = x^2$
$y = x^2 + 2$
$y = x^2 - 4$

2. Find $\frac{dy}{dx}$ for each

$\frac{dy}{dx} = 2x$

3. Find slope for each x value

	m
$x = -2$	-4
$x = -1$	-2
$x = 0$	0
$x = 1$	2
$x = 2$	4

$\left.\frac{dy}{dx}\right|_{x=-2} = 2(-2)$

→ Find $\frac{dy}{dx}$

A) $x^2 + y^2 = 4$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{2y}{2y}\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

B) $2x^2 + 3y = 4$

$4x + 3\frac{dy}{dx} = 0$

$\frac{3\frac{dy}{dx}}{3} = \frac{-4x}{3}$

$\frac{dy}{dx} = \frac{-4x}{3}$

C) $x + y^2 = 7$

$1 + 2y\frac{dy}{dx} = 0$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-1}{2y}$

$\frac{dy}{dx} = \frac{-1}{2y}$

Find Slopes

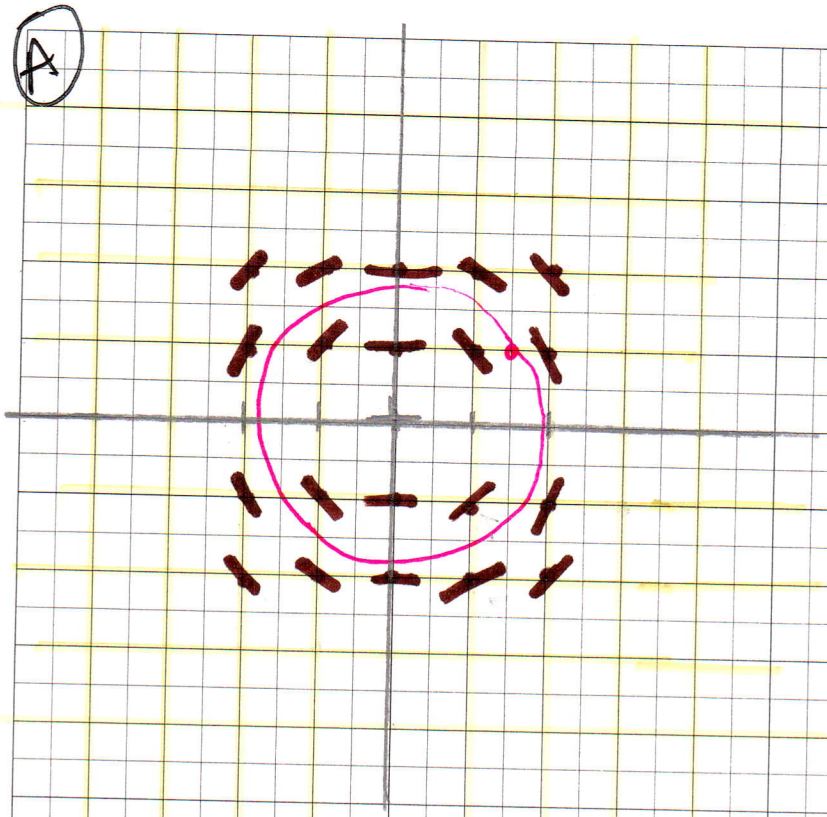
A) $\frac{dy}{dx} = \frac{-x}{y}$

x	y	$\frac{dy}{dx} = \frac{-x}{y}$
0	-2	0
0	-1	0
0	1	0
0	2	0
1	-2	1/2
1	-1	1
1	1	-1
1	2	-1/2
-1	-2	-1/2
-1	-1	-1
-1	1	1
-1	2	1/2
2	-2	1
2	-1	2
2	1	-2
2	2	-1
-2	-2	-1
-2	-1	-2
-2	1	2
-2	2	1

B) $\frac{dy}{dx} = \frac{-4}{3}x$

x	$\frac{dy}{dx}$
-3	4
-2	$8/3$
-1	$4/3$
0	0
1	$-4/3$
2	$-8/3$
3	-4

(A)

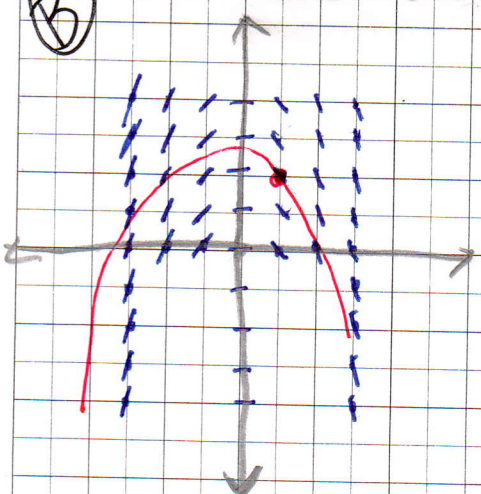


draw a solution curve through the point (1.5, 1)

$x^2 + y^2 = 4$ ← Shape made by "$\underbrace{\text{solution curves}}_{\text{Circles}}$"

$$\frac{dy}{dx} = \frac{-x}{y}$$

(B)



draw a solution curve through (2, 1)

$2x^2 + 3y = 4$

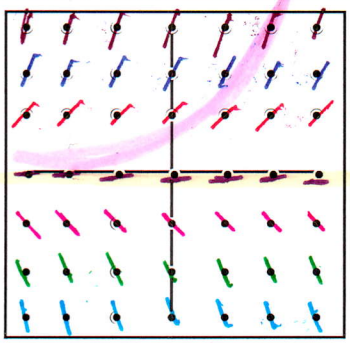
$$\frac{dy}{dx} = \frac{-4x}{3}$$

Slope Fields - Classwork

In this section, we can solve differential equations by obtaining a *slope field* (sometimes called a *direction field*) that approximates the general solution. The slope field of a first-order differential equation says that the differential equation can be interpreted as a statement about the slopes of its solution curves. You are given dy/dx.

1) $\frac{dy}{dx} = y$. Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3	-3	-2	-1	0	1	2	3
2	-3	-2	-1	0	1	2	3
1	-3	-2	-1	0	1	2	3
0	-3	-2	-1	0	1	2	3
-1	-3	-2	-1	0	1	2	3
-2	-3	-2	-1	0	1	2	3
-3	-3	-2	-1	0	1	2	3

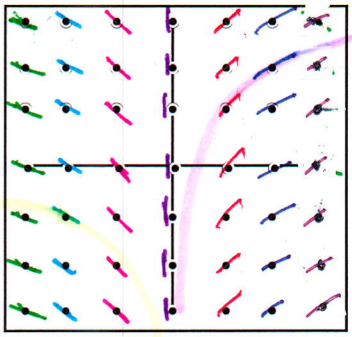


- Slopes are positive when $y > 0$
- Slopes are negative when $y < 0$
- Solution curves look like e^x

Find the solution going through
a) (1, 1) b) (-2, 0)

2) $\frac{dy}{dx} = \frac{1}{x}$ Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3	-1/3	-1/3	-1/3	-1/3	-1/3	-1/3	-1/3
2	-1/2	-1/2	-1/2	-1/2	-1/2	-1/2	-1/2
1	-1	-1	-1	-1	-1	-1	-1
0	Und	Und	Und	Und	Und	Und	Und
+1	1	1	1	1	1	1	1
+2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
+3	1/3	1/3	1/3	1/3	1/3	1/3	1/3



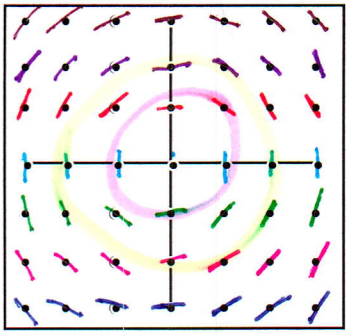
- Slopes are pos. when $x > 0$.
- Slopes are neg when $x < 0$
- Soln curves look like $\ln x$

Find the solution going through
a) (1, 1) b) (-2, -1)

3) $\frac{dy}{dx} = \frac{-x}{y}$ Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3	1	3/2	3	Und	-3	-3/2	-1
2	2/3	1	2	Und	-2	-1	-2/3
1	1/3	1/2	1	Und	-1	-1/2	-1/3
0	0	0	0	?	0	0	0
-1	-1/3	-1/2	-1	Und	1	1/2	1/3
-2	-2/2	-1	-2	Und	2	1	2/3
-3	1	-3/2	-3	Und	3	3/2	1

$\frac{0}{0}$ indeterm.

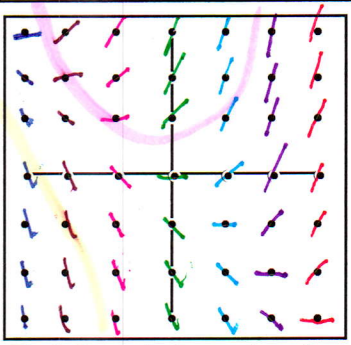


- Slopes are pos when $x > 0$ + $y < 0$ and $x < 0$ + $y > 0$
- Slopes are neg when $x > 0$ + $y > 0$ and $x < 0$ + $y < 0$
- Solution curves look like circles

Find the solution going through
a) (1, 1) b) (-2, 0)

4) $\frac{dy}{dx} = x + y$ Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3	0	1	2	3	4	5	6
2	-1	0	1	2	3	4	5
1	-2	-1	0	1	2	3	4
0	-3	-2	-1	0	1	2	3
-1	-4	-3	-2	-1	0	1	2
-2	-5	-4	-3	-2	-1	0	1
-3	-6	-5	-4	-3	-2	-1	0



- Slopes are pos when $x > -y$ or $y > -x$
- Slopes are neg when $x < -y$ or $y < -x$

Find the solution going through
a) (1, 1) b) (-2, -1)