AP Calculus
21 Estimating Area with Finite Sums

Name: Key
Date: ____________ Per: ______

21 Estimating Area with Finite Sums

Work on this sheet of paper

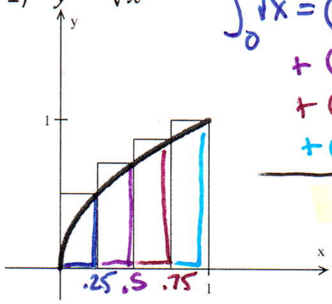
Use upper and lower sums to approximate the area of the region using the indicated number of subintervals (of equal length).

1) $y = \sqrt{x}$

base · height

$\int_0^1 \sqrt{x} = (.25)f(.25) + (.25)f(.5) + (.25)f(.75) + (.25)f(1)$

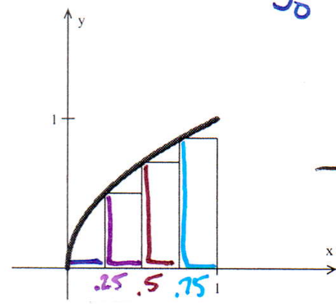
$= .76828$



b · h

$\int_0^1 \sqrt{x} \approx (.25)f(0) + (.25)f(.25) + (.25)f(.5) + (.25)f(.75)$

$= .51828$



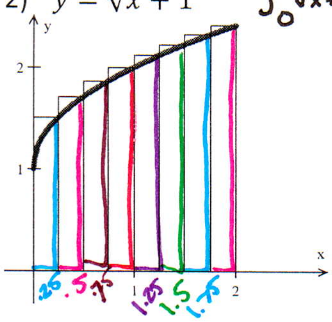
2) $y = \sqrt{x} + 1$

$\frac{2}{8} = \frac{1}{4}$

b · h

$\int_0^2 \sqrt{x}+1 \approx (.25)f(.25) + (.25)f(.5) + (.25)f(.75) + (.25)f(1) + (.25)f(1.25) + (.25)f(1.5) + (.25)f(1.75) + (.25)f(2)$

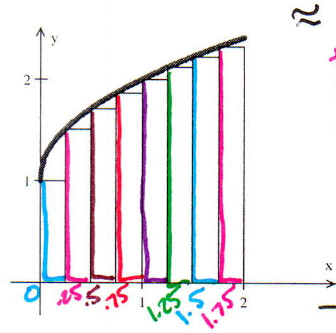
$= 4.03825$



b · h

$\approx (.25)f(0) + (.25)f(.25) + (.25)f(.5) + (.25)f(.75) + (.25)f(1) + (.25)f(1.25) + (.25)f(1.5) + (.25)f(1.75)$

$= 3.68470$



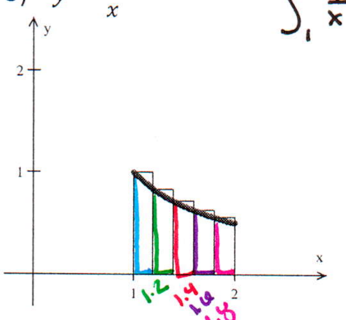
3) $y = \frac{1}{x}$

$\frac{1}{5} = .2$

b · h

$\int_1^2 \frac{1}{x} \approx (.2)f(1) + (.2)f(1.2) + (.2)f(1.4) + (.2)f(1.6) + (.2)f(1.8)$

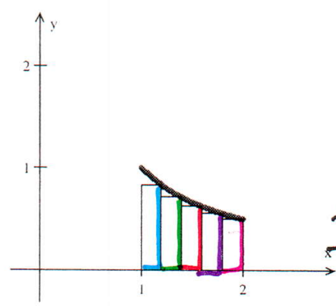
$= 0.74563$



b · h

$(.2)f(1.2) + (.2)f(1.4) + (.2)f(1.6) + (.2)f(1.8) + (.2)f(2)$

$= 0.64563$



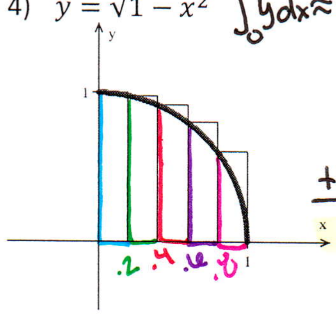
4) $y = \sqrt{1 - x^2}$

$\frac{1}{5} = .2$

b · h

$\int_0^1 y\,dx \approx (.2)f(0) + (.2)f(.2) + (.2)f(.4) + (.2)f(.6) + (.2)f(.8)$

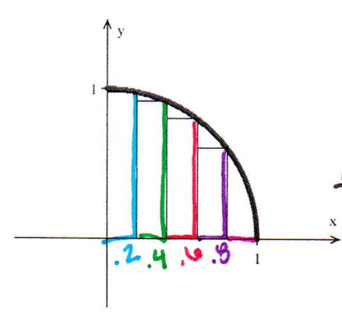
$= 0.85926$



b · h

$(.2)f(.2) + (.2)f(.4) + (.2)f(.6) + (.2)f(.8) + (.2)f(1)$

$= 0.65926$

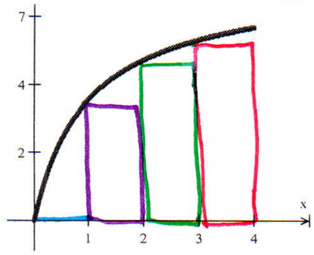


5) Consider the region bounded by the graphs of $f(x) = \frac{8x}{x+1}$, $x = 0$, $x = 4$ and $y = 0$ as shown in the figure.

a. Redraw the figure, and complete and shade the rectangles representing the lower sum when $n = 4$. Find this lower sum.

$$\int_0^4 \frac{8x}{x+1}\,dx \approx (1)f(0) + (1)f(1) + (1)f(2) + (1)f(3)$$

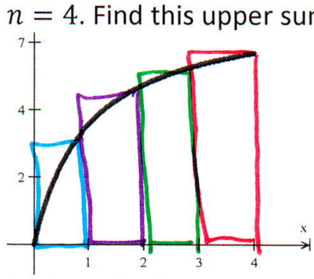
$$= 46/3 \approx 15.33333$$



b. Redraw the figure, and complete and shade the rectangles representing the upper sum when $n = 4$. Find this upper sum.

$$\int_0^4 \frac{8x}{x+1}\,dx \approx (1)f(1) + (1)f(2) + (1)f(3) + (1)f(4)$$

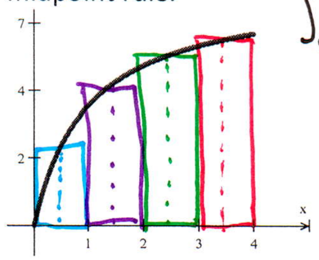
$$= \frac{326}{15} \approx 21.73333$$



c. Redraw the figure, and complete and shade the rectangles whose heights are determined by the functional values at the midpoint of each subinterval when $n = 4$. Find this sum using the midpoint rule.

$$\int_0^4 \frac{8x}{x+1}\,dx \approx (1)f(.5) + (1)f(1.5) + (1)f(2.5) + (1)f(3.5)$$

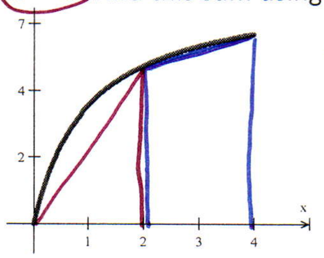
$$= 19.40317$$



d. Redraw the figure, and complete and shade the trapezoids representing the subintervals when $n = 2$. Find this sum using the trapezoidal rule.

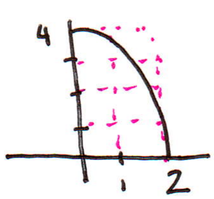
$$\int_0^4 \frac{8x}{x+1}\,dx \approx \frac{f(0)+f(2)}{2}\cdot 2 + \frac{f(2)+f(4)}{2}\cdot 2$$

$$= \frac{256}{15} \approx 17.06667$$



6) Which of the value best approximates the area of the region between the x-axis and the graph of $f(x) = 4 - x^2$ on the interval $[0,2]$? Make your selection on the basis of a sketch of the region and NOT by performing calculations.

(A) -2
(B) 6
(C) 10
(D) 3
(E) 8



← approx 8 squares

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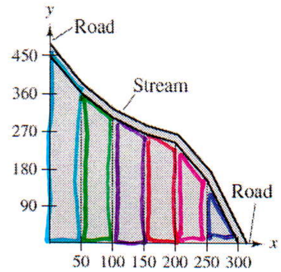
7) The table lists the measurements of a lot bounded by a stream and two straight roads that meet at a right angles, where x and y are measured in feet. Use the trapezoidal method to find the area of the lot.

x	0	50	100	150	200	250	300
y	450	362	305	268	245	156	0

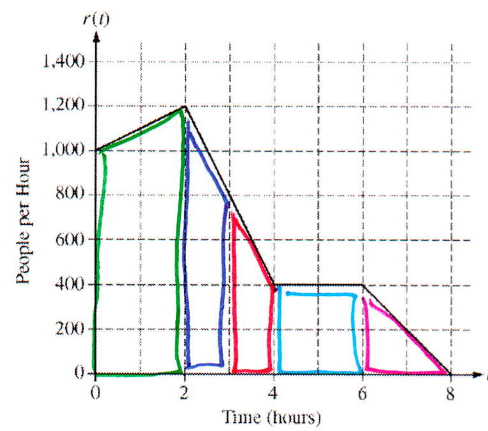


$$\frac{450+362}{2}\cdot 50+\frac{362+305}{2}\cdot 50+\frac{305+268}{2}\cdot 50$$

$$+\frac{268+245}{2}\cdot 50+\frac{245+156}{2}\cdot 50+\frac{156+0}{2}\cdot 50$$

$$= 78{,}050$$

8) What is the area under the curve from
a. $t = 0 \; to \; t = 3$?
b. $t = 0 \; to \; t = 8$?

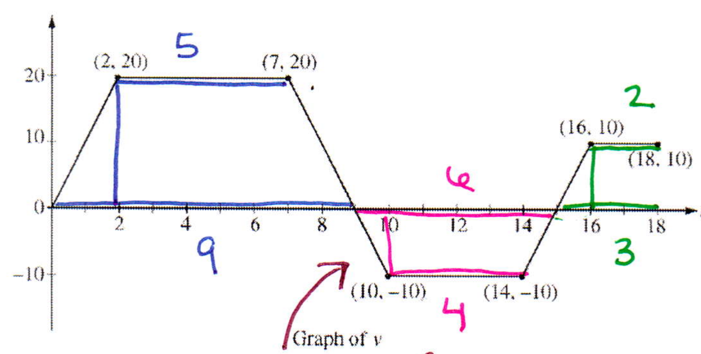


a) $\frac{1000+1200}{2}\cdot 2 + \frac{1200+800}{2}\cdot 1 = 3200$

b) $3200 + \frac{800+400}{2}\cdot 1 + 400\cdot 2 + \frac{1}{2}(2)(400)$

$= 5{,}000$

9) What is the area under the curve from
a. $t = 0 \; to \; t = 18$?
b. What is the TOTAL area (unsigned) from $t = 0 \; to \; t = 18$?



negative for a
positive for b

b) $\frac{9+5}{2}\cdot 20 + \frac{6+4}{2}\cdot 10 + \frac{3+2}{2}\cdot 10$

$= 140 + 50 + 25$

$= 215$

a) $140 - 50 + 25$

$= 115$

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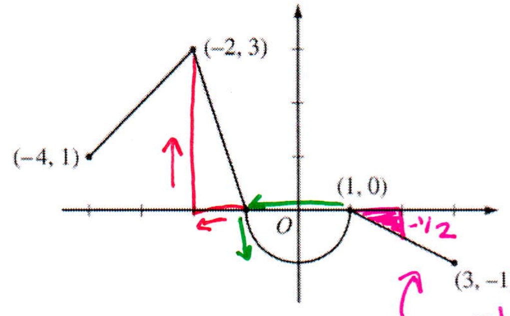
10) Find the area under the curve from
   a. $t = 1$ to $t = 2$?
   b. $t = 1$ to $t = -2$ (Consider the direction)



Graph of f

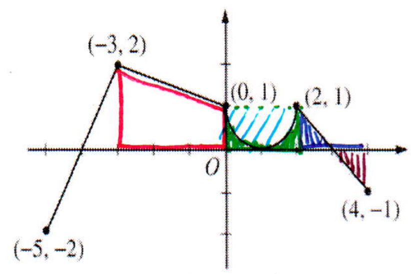
a. $\frac{1}{2}b \cdot h = \frac{1}{2}(1)(-\frac{1}{2}) = \frac{-1}{4}$

b. $\frac{1}{2}\pi r^2 \rightarrow \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ (+ since $\leftarrow \downarrow$)

$\frac{1}{2}b \cdot h \rightarrow \frac{1}{2}(-1)(3) = \frac{-3}{2}$

$\frac{\pi}{2} + \frac{-3}{2} = \frac{\pi - 3}{2}$

11) Find the area under the curve from
   a. $t = -3$ to $t = 0$?
   b. $t = 0$ to $t = 2$?
   c. $t = 2$ to $t = 4$?



Graph of f

a) $\frac{2+1}{2} \cdot 3 = \frac{9}{2}$

b) $b \cdot h - \frac{1}{2}\pi r^2$

$2 \cdot 1 - \frac{1}{2}\pi(1)^2 = 2 - \frac{\pi}{2}$

c) $\frac{1}{2}b \cdot h + \frac{1}{2}b \cdot h$

$\frac{1}{2} \cdot 1 \cdot 1 + \frac{1}{2}(1)(-1)$

$\frac{1}{2} + \frac{-1}{2} = 0$