

2.1 Estimating Area with Finite Sums

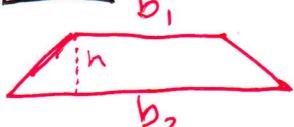
Geometry Formulas

$$A = \pi r^2$$

$$A = \frac{1}{2} \pi r^2$$

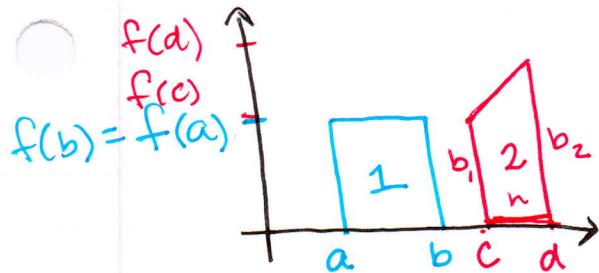

$$A = b \cdot h$$

$$A = \frac{1}{2} b \cdot h$$

$$A = \frac{(b_1 + b_2)}{2} \cdot h$$


b + h must be \perp

In the Coordinate Plane



$$\text{Area of 1} = \underbrace{(b-a)}_{\text{base}} \underbrace{f(a)}_{\text{height}}$$

Left hand
right hand

$$= \underbrace{(b-a)}_{\text{base}} \underbrace{f(b)}_{\text{height}}$$

$$\text{Area of 2} = \frac{(f(c) + f(d))}{2} \cdot (d-c)$$

$\underbrace{(f(c) + f(d))}_{\text{bases}}$ $\underbrace{(d-c)}_{\text{height}}$

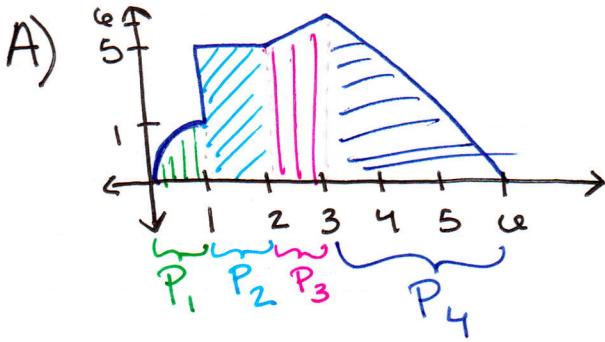
Partition - Split into parts that can "easily" be made into nice geometric shapes

Directed Area - $\rightarrow \uparrow$ are positive measures
(Signed Area) $\leftarrow \downarrow$ are negative measures

* $f'(x)$ means "slope"

$\int_a^b f(x) dx$ means the area from $x=a$ to $x=b$ between the curve $f(x)$ and the x -axis

Example 1: Create a partition and find the area represented



$$P_1 = \frac{1}{4}\pi(1)^2 = \pi/4$$

$$P_2 = (1)(5) = 5 = 10/2$$

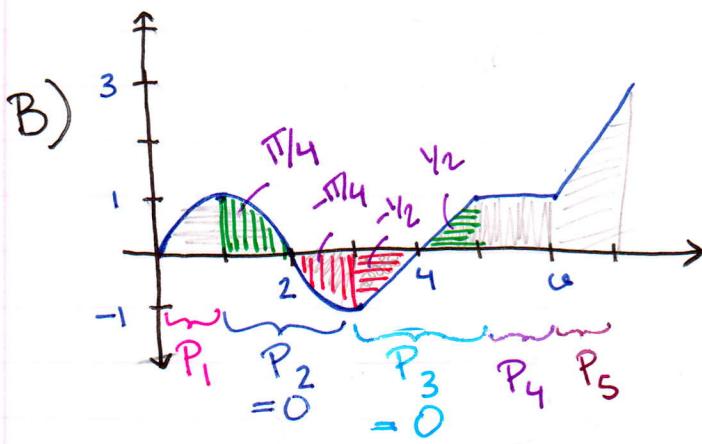
$$P_3 = \frac{(5+6)}{2} \cdot (1) = \frac{11}{2}$$

$$P_4 = \frac{1}{2}(3)(6) = 9 = 18/2$$

$$\int_0^6 f(x) dx = P_1 + P_2 + P_3 + P_4 = \frac{\pi}{4} + \frac{39}{2}$$

$$\int_2^6 f(x) dx = P_3 + P_4 = 29/2$$

$$\int_3^6 f(x) dx = -\frac{21}{2}$$



$$P_1 = \pi/4$$

$$P_4 = 1$$

$$P_5 = \frac{(1+3)}{2} \cdot 1 = 2$$

$$\int_0^7 f(x) dx = \frac{\pi}{4} + 1 + 2 = 3 + \pi/4$$

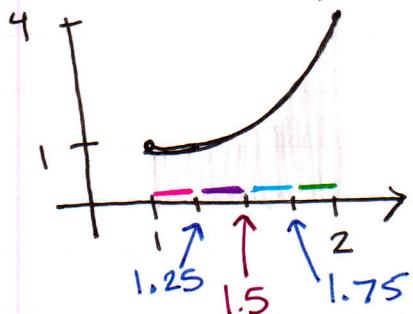
rectangle - left, right
or midpoint

Riemann Sums + Trapezoidal Rule

Estimate $\int_1^2 x^2$

w/ $n = 4$

\leftarrow number of partitions



a) Using a left hand Riemann Sum



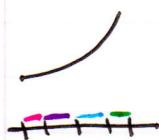
$$\int_1^2 x^2 \approx (.25)f(1) + (.25)f(1.25) + (.25)f(1.5) + (.25)f(1.75)$$

$$(.25) \cdot 1 + (.25)(1.56) + (.25)(2.25) + (.25)(3.06)$$

$$.25 + .39 + .56 + .76$$

$$= 1.96$$

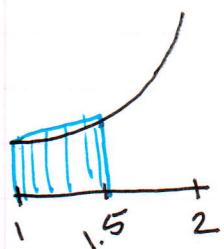
b) Using a right hand Riemann Sum



$$\int_1^2 x^2 \approx (.25)f(1.25) + (.25)f(1.5) + (.25)f(1.75) + (.25)f(2)$$

$$= 2.718$$

c) Using the trapezoid rule w/ $n = 2$



$$\int_1^2 x^2 \approx \underbrace{\left(\frac{f(1) + f(1.5)}{2} \right)}_{\text{bases}} (.5) + \underbrace{\left(\frac{f(1.5) + f(2)}{2} \right)}_{\text{height}} (.5)$$

$$.8125 + 1.5625$$

$$= 2.375$$