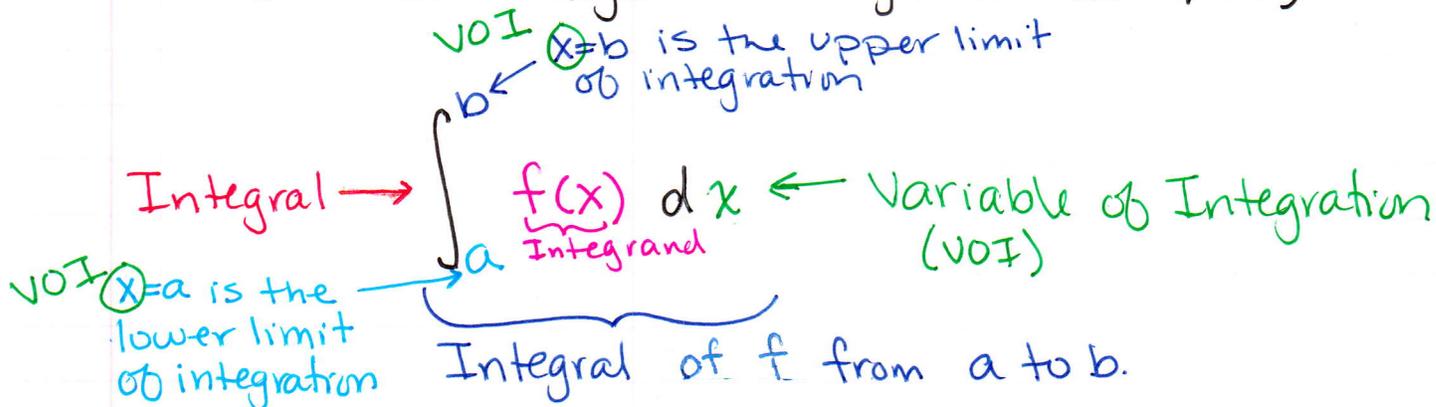


22 Definite Integrals (assign for Fri w/sub)



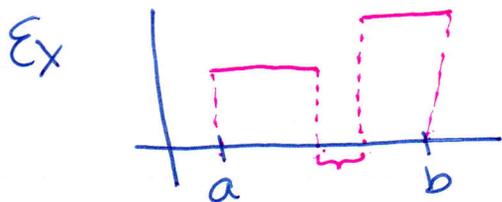
Recall: Inverse - Undo

Derivative - $y = f(x)$
 $\frac{dy}{dx} = f'(x)$

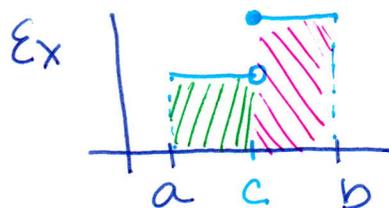
Integral - Inverse of derivatives

$$dy = f'(x) \cdot dx$$

To be Integrable a fn must be cts



not integrable
 b/c can't find area
 w/ no height



IS Integrable b/c
 we can break it up
 (Partition) cts pieces

Properties of Integrals

f is cts on $[a, b]$

1. $\int_a^a f(x) dx = 0$

2. $\int_a^b f(x) dx = - \int_b^a f(x) dx$ (Signed area)

3. $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

* c is between a + b

4. $\int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx$

* k is a constant

$$\frac{d}{dx} [c \cdot f(x)] = c \cdot \frac{d}{dx} [f(x)]$$

$$3 \cdot x^2 \rightarrow 3 \cdot (2x^1)$$

5. $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

Example 1 $\int_0^5 f(x) dx = 10$ $\int_5^7 f(x) dx = 3$

A) $\int_0^7 f(x) dx = \int_0^5 f(x) dx + \int_5^7 f(x) dx = 13$

B) $\int_5^0 f(x) dx = -10$

C) $\int_5^5 f(x) dx = 0$

D) $\int_0^5 3 \cdot f(x) dx = 3(10) = 30$

Example 2 $\int_2^6 f(x) dx = 10$ $\int_2^6 g(x) dx = -2$

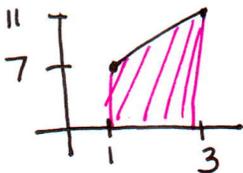
A) $\int_2^6 [f(x) + g(x)] dx = 10 + (-2) = 8$

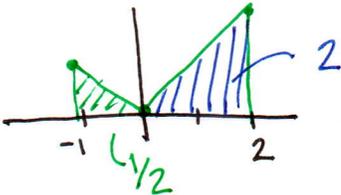
B) $\int_2^6 [g(x) - f(x)] dx = -2 - 10 = -12$

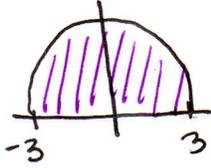
C) $\int_2^6 [2 \cdot g(x) + 3 \cdot f(x)] dx = 2(-2) + 3(10) = 26$

D) $\int_2^6 [3 \cdot g(x) - 4 \cdot f(x)] dx = - \left(\underbrace{3(-2) - 4(10)}_{-46} \right) = 46$
 OR $= 3(2) - 4(10) = 46$

Example 3 Sketch a graph + use geometric figures to evaluate the integral.

A) $\int_1^3 (2x + 5) dx = 18$  $\frac{7+11}{2} \cdot 2$

B) $\int_{-1}^2 |x| dx = 2\frac{1}{2}$ 

C) $\int_{-3}^3 \sqrt{9-x^2} dx = \frac{9\pi}{2}$  $\frac{1}{2} (\pi 3^2)$
 $= 4.5\pi$