

* due mon 4/3 Integrals from FRONT of BOOK

$$\int_a^b f(x) dx = \boxed{x + C}$$

23 Antiderivatives and Indefinite Integrals Synonyms

Find y

$$1. y = 3x^2 + 2x + 7$$

$$y' = 6x + 2$$

$$2. y = \sin x + 3$$

$$y' = \cos x$$

$$3. y = \ln |x^3 + 2| + 9$$

$$y' = \frac{3x^2}{x^3 + 2} = \frac{u'}{u}$$

f'

$F(x)$ is an Antiderivative (vs the antider.) of f ,

plural

singular

so $G(x) = F(x) + C$ represents the family of all
antiderivatives of f .



Differentiation



$f(x)$

$f'(x)$

$f''(x)$

Integration



$F(x)$

$F'(x)$

$F''(x)$

$$* F'(x) = f(x)$$

$\sqrt{} + (\)^2$ are inverses

$\frac{d}{dx} []$ and $\int dx$ are inverses

$$* f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f'(a)$ - slope of f at $x=a$

vs

$f'(x)$ - egn for all slopes
of f

short cut

For Integrals

learn shortcut

long way using
geometric figures
to find area.

$\int_a^b f(x) dx$
- # value for
a finite area

vs $\int f(x) dx$
- equation to
represent
all areas

Slopefields
graphical representation
of a solution, particular solution

Solving a Differential Equation

1. Separate the variables
2. Integrate both sides
3. Constant of Integration (only one side, DON'T FORGET)

Ex 1

$$y' = 2x$$

$$dx \cdot \frac{dy}{dx} = 2x \cdot dx$$

variable
of
integration

$$\int 1 \cdot dy = \int 2x \cdot dx$$

$$y + C_1 = x^2 + C_2 \quad \Rightarrow \quad y = x^2 + C$$

Ex 2 Evaluate each indefinite integral

$$\begin{aligned} A) \int (12s^2 + 1) ds &= \frac{12s^3}{3} + s + C \\ &= 4s^3 + s + C \end{aligned}$$

$$\begin{aligned} B) \int (5x^4 + 16x^3) dx &= \frac{5x^5}{5} + \frac{16x^4}{4} + C \\ &= x^5 + 4x^4 + C \end{aligned}$$

$$\begin{aligned} C) \int -8r^1 - 5 dr &= \frac{-8r^2}{2} - 5r + C \\ &= -4r^2 - 5r + C \end{aligned}$$

$$\begin{aligned} D) \int 20s^4 - 20s^3 - 3s + 7 ds \\ = 4s^5 - 5s^4 - \frac{3s^2}{2} + 7s + C \end{aligned}$$

$$\frac{d}{dx} [x^{-1}] = -(-1)x^{-2}$$

$$= x^{-2}$$

E) $\int x^{-2} dx - \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$

F) $\int x^{\frac{2}{3}} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5} x^{\frac{5}{3}} + C$

G) $\int x^{-1} dx = \frac{x^0}{0}$ $x \neq 0$ und
 $= \int \frac{1}{x} dx = \ln|x| + C$

H) $\int -\frac{5}{x} dx = -5 \underbrace{\int \frac{1}{x} dx}_{ab+c/a(b+c)}$
 $= -5 \cdot (\ln|x| + C)$
 $= -5 \cdot \ln|x| + C$

I) $\int \underbrace{2 \sec(x) \cdot \tan(x)}_{ab+c/a(b+c)} dx = 2 \sec x + C$

J) $\int 2 \csc^2 x dx = -2 \cot x + C$

K) $\int -5 \sec x \cdot \tan x + \frac{3}{x} - 3x^7 dx$
 $= -5 \sec x + 3 \ln|x| - \frac{3}{8}x^8 + C$

L) $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$

M) $\int \frac{1}{|b|\sqrt{b^2-1}} db = \sec^{-1}(b) + C$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$N) \int \frac{2 dx}{\sqrt{9 - 4x^2}} = \sin^{-1}\left(\frac{2x}{3}\right) + C$$

$\begin{array}{l} a=3 \\ a^2=9 \\ \uparrow \\ 9=a^2 \end{array}$ $\begin{array}{l} u^2=4x^2 \\ \uparrow \\ 4x^2=u^2 \end{array}$ $\begin{array}{l} u=2x \\ du=2 \cdot dx \\ \frac{du}{dx}=2 \end{array}$