AP Calculus　　　　　　　　　　Name Stew Dent

24 Fundamental Theorem of Calculus

Date________________ Period____

Evaluate each definite integral.

1) $\int_{3}^{6} (x^2 - 8x + 18)\, dx = \frac{1}{3}x^3 - 4x^2 + 18x \Big]_{3}^{6}$

9

$= 36 - 27$

$= 9$

2) $\int_{-4}^{-1} (-x^2 - 6x - 5)\, dx = -\frac{1}{3}x^3 - 3x^2 - 5x \Big]_{-4}^{-1}$

9

$= \frac{7}{3} - \frac{-20}{3}$

$= \frac{27}{3}$

$= 9$

3) $\int_{-5}^{-2} \left(-\frac{x^2}{2} - 4x - 8\right) dx = -\frac{1}{6}x^3 - 2x^2 - 8x \Big]_{-5}^{-2}$

$-\frac{3}{2} = -1.5$

$= \frac{28}{3} - \frac{65}{6}$

$= \frac{56}{6} - \frac{65}{6} = \frac{-9}{6}$

$= -3/2$

4) $\int_{-5}^{0} (-x + 2)\, dx = -\frac{1}{2}x^2 + 2x \Big]_{-5}^{0}$

$\frac{45}{2} = 22.5$

$= 0 - \frac{-45}{2}$

$= \frac{45}{2}$

5) $\int_{2}^{5} -4x^{\frac{1}{3}}\, dx = -3x^{4/3} \Big]_{2}^{5}$

$-15\sqrt[3]{5} + 6\sqrt[3]{2} \approx -18.09$

$-3(5)^{3/3}(5)^{1/3} - -3(2)^{3/3}(2)^{1/3}$

$= -15\sqrt[3]{5} + 6\sqrt[3]{2}$

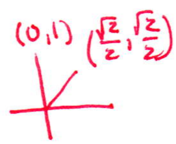
$-4x^{-2}$

6) $\int_{1}^{3} -\frac{4}{x^2}\, dx = \frac{+4}{x} \Big]_{1}^{3} = \frac{4}{3} - \frac{4}{1}$

$-\frac{8}{3} \approx -2.667$ =

$= \frac{4}{3} - \frac{12}{3}$

$= \frac{-8}{3}$



7) $\int_{-2}^{0} -2e^x\, dx = -2e^x \Big]_{-2}^{0}$

$\frac{-2e^2 + 2}{e^2} \approx -1.729$

$= -2 - -2e^{-2}$

$= \frac{-2e^2}{e^2} + \frac{2}{e^2}$

$= \frac{-2e^2 + 2}{e^2}$

8) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\csc x \cdot \cot x\, dx \quad -2\csc x \Big]_{\pi/4}^{\pi/2}$

$-2 + 2\sqrt{2} \approx 0.828$

$= -2(1 - \sqrt{2})$

$= -2 + 2\sqrt{2}$

For each problem, find $F'(x)$.

9) $F(x) = \int_2^x (-t + 2)\, dt$

$F'(x) = -x + 2$

10) $F(x) = \int_{-1}^x (2t - 1)\, dt$

$F'(x) = 2x - 1$

11) $F(x) = \int_{-4}^x -\frac{4}{t^3}\, dt$

$F'(x) = -\frac{4}{x^3}$

12) $F(x) = \int_{-5}^x -\frac{2}{(t+3)^3}\, dt$

$F'(x) = -\frac{2}{(x+3)^3}$

For each problem, find the average value of the function over the given interval.

f(c)

13) $f(x) = x$; $[-6, -4]$

-5

$$\frac{1}{-4 - -6} \int_{-6}^{-4} x\, dx = \frac{1}{2}\left[\frac{1}{2}x^2\right]_{-6}^{-4}$$
$$= \frac{1}{4} \cdot x^2\Big]_{-6}^{-4}$$
$$= \frac{1}{4}\left[16 - 36\right]$$
$$= \frac{1}{4} \cdot 20 = \boxed{-5}$$

14) $f(x) = -\frac{x^2}{2} - 3x + \frac{1}{2}$; $[-5, -1]$

$\frac{13}{3} \approx 4.333$

$$\frac{1}{-1 - -5} \int_{-5}^{-1} -\frac{x^2}{2} - 3x + \frac{1}{2}$$
$$= \frac{1}{4}\left[-\frac{x^3}{6} - \frac{3x^2}{2} + \frac{1}{2}x\right]_{-5}^{-1}$$
$$= \frac{1}{4}\left[\frac{-11}{6} - \frac{-115}{6}\right] = \frac{13}{3}$$

15) $f(x) = -x^3 + 2x^2 + 2$; $[0, 3]$

$\frac{5}{4} = 1.25$

$$\frac{1}{3-0} \int_0^3 -x^3 + 2x^2 + 2\, dx$$
$$= \frac{1}{3}\left[-\frac{x^4}{4} + \frac{2}{3}x^3 + 2x\right]_0^3$$
$$= \frac{1}{3}\left[\frac{15}{4} - 0\right]$$
$$= \frac{5}{4}$$

16) $f(x) = -x^3 + 3x^2 - 3$; $[1, 3]$

0

$$\frac{1}{3-1} \int_1^3 -x^3 + 3x^2 - 3\, dx$$
$$= \frac{1}{2}\left[-\frac{x^3}{4} + x^3 - 3x\right]_1^3$$
$$= \frac{1}{2}\left[\frac{-9}{4} - \frac{-9}{4}\right]$$
$$= 0$$