Calculus Name Key

#27 U-Subsitution

Date________ Period____

Evaluate each indefinite integral. Use the provided substitution.

1) $\int (5x^2+3)^5 \cdot 30x\,dx;\ u = 5x^2+3$

$3 \cdot 10x\,dx$

$du = 10x\,dx$

$3\int u^5\,du$

$= 3\frac{u^6}{6} + C$

$\boxed{= \frac{(5x^2+3)^6}{2} + C}$

2) $\int 75x^4(3x^5-5)^4\,dx;\ u = 3x^5-5$

$5 \cdot 15x^4\,dx$

$du = 15x^4\,dx$

$5\int u^4\,du$

$= \frac{5u^5}{5} + C$

$\boxed{= (3x^5-5)^5 + C}$

Evaluate each indefinite integral.

3) $\int 75x^2(5x^3+2)^5\,dx$

$5 \cdot 15x^2$

$u = 5x^3+2$

$du = 15x^2\,dx$

$5\int u^5\,du$

$= \frac{5u^6}{6} + C$

$\boxed{\frac{5(5x^3+2)^6}{6} + C}$

4) $\int 30x^4(3x^5-1)^3\,dx$

$2 \cdot 15x^4$

$u = 3x^5-1$

$du = 15x^4\,dx$

$= 2\int u^3\,du$

$= \frac{2u^4}{4} + C$

$\boxed{= \frac{(3x^5-1)^4}{2} + C}$

Evaluate each indefinite integral. Use the provided substitution.

5) $\int \frac{5(-4+\ln -3x)^5}{x}\,dx;\ u = -4+\ln(-3x)$

$\ln u \rightarrow \frac{u'}{u}$

$du = \frac{1}{-3x} \cdot -3\,dx$

$du = \frac{1}{x}\,dx$

$= 5\int u^5\,du$

$= \frac{5u^6}{6} + C$

$\boxed{= \frac{5(-4+\ln(-3x))^6}{6} + C}$

6) $\int \frac{5(2+\ln -3x)^5}{x}\,dx;\ u = 2+\ln(-3x)$

$du = \frac{1}{-3x} \cdot -3\,dx = \frac{1}{x}\,dx$

$= 5\int u^5\,du$

$= \frac{5u^6}{6} + C$

$\boxed{= \frac{5(2+\ln(-3x))^6}{6} + C}$

Evaluate each indefinite integral.

7) $\displaystyle\int \frac{2(3+\ln 3x)^3}{x}\,dx$

$u = 3+\ln 3x$

$du = \frac{1}{3x}\cdot 3\,dx$

$du = \frac{1}{x}\,dx$

$= 2\int u^3\,du$

$= \frac{2u^4}{4} + C$

$= \boxed{\frac{(3+\ln 3x)^4}{2} + C}$

8) $\displaystyle\int \frac{3(-3+\ln 3x)^4}{x}\,dx$

$u = -3+\ln 3x$

$du = \frac{1}{3x}\cdot 3\,dx$

$du = \frac{1}{x}\,dx$

$= 3\int u^4\,du$

$= \frac{3u^5}{5} + C$

$= \boxed{\frac{3(-3+\ln 3x)^5}{5} + C}$

Evaluate each indefinite integral. Use the provided substitution.

9) $\displaystyle\int \cancel{10}^{\,2\cdot 5} \cdot \csc^2 -5x \cdot \underbrace{\cot^3 -5x}_{u^3}\,dx$; $u = \cot(-5x)$

$du = -\csc^2(-5x)\cdot -5dx$

$du = 5\csc^2(-5x)dx$

$= 2\int u^3\,du$

$= \frac{2u^4}{4} + C$

$= \boxed{\frac{\cot^4(-5x)}{2} + C}$

10) $\displaystyle\int \cancel{-8}^{\,4\cdot(-2)}\sin 2x \cdot \underbrace{\cos^3 2x}_{u^3}\,dx$; $u = \cos 2x$

$du = -\sin(2x)\cdot 2\cdot dx$

$du = -2\sin(2x)dx$

$= 4\int u^3\,du$

$= \frac{4u^4}{4} + C$

$= \boxed{\cos^4(2x) + C}$

Evaluate each indefinite integral.

11) $\displaystyle\int \cancel{4}^{\,2\cdot 2}\cos 2x \cdot \overbrace{\sin^5 2x}^{(\sin 2x)^5}\,dx$

$u = \sin 2x$

$du = \cos(2x)\cdot 2\,dx$

$= 2\int u^5\,du$

$= \frac{2u^6}{6} + C$

$= \boxed{\frac{\sin^6(2x)}{3} + C}$

12) $\displaystyle\int \cancel{-10}^{\,2\cdot(-5)} \cdot \sec^2 -5x \cdot \overbrace{\tan^3 -5x}^{[\tan(-5x)]^3}\,dx$

$u = \tan(-5x)$

$du = \sec^2(-5x)\cdot -5dx$

$= 2\int u^3\,du$

$= \frac{2u^4}{4} + C$

$= \boxed{\frac{\tan^4(-5x)}{2} + C}$

Express each definite integral in terms of u, but do not evaluate.

13) $\int_{-1}^{0} -12x(2x^2-2)^2\,dx;\ u = 2x^2-2$

$-3 \cdot 4$

$du = 4x\,dx$

$= -3\int_{0}^{-2} u^2\,du$

$u(0) = 2(0)^2 - 2 = -2$

$u(-1) = 2(-1)^2 - 2$

$= 0$

14) $\int_{-1}^{0} \frac{18x}{(3x^2+3)^2}\,dx;\ u = 3x^2+3$

$3 \cdot 6$

$du = 6x\,dx$

$= 3\int_{6}^{3} \frac{1}{u^2}\,du$

$u(0) = 3(0)^2+3$

$u(-1) = 3(-1)^2+3$

15) $\int_{-1}^{0} -\frac{18x}{(3x^2+1)^2}\,dx;\ u = 3x^2+1$

$3 \cdot 6$

$du = 6x\,dx$

$-3\int_{4}^{1} \frac{1}{u^2}\,du$

$u(0) = 3(0)^2+1$

$u(-1) = 3(-1)^2+1$

16) $\int_{0}^{3} \frac{6x}{(x^2+3)^2}\,dx;\ u = x^2+3$

$3 \cdot 2$

$du = 2x\,dx$

$= 3\int_{3}^{12} \frac{1}{u^2}\,du$

$u(3) = 3^2+3$

$u(0) = 0^2+3$

Evaluate each definite integral.

17) $\int_{-1}^{0} \frac{24x}{(4x^2+2)^2}\,dx;\ u = 4x^2+2$

$3 \cdot 8$

$du = 8x\,dx$

$= 3\int_{6}^{2} \frac{1}{u^2}\,du$

$u(0) = 4(0)^2+2$

$u(-1) = 4(-1)^2+2$

$= \frac{3u^{-1}}{-1}\Big]_{6}^{2} = -3\left(\frac{1}{2} - \frac{1}{6}\right)$

$= -3\left(\frac{3}{6} - \frac{1}{6}\right)$

$= -3\left(\frac{2}{6}\right)$

$= \boxed{-1}$

18) $\int_{0}^{1} \frac{4x}{(2x^2+1)^2}\,dx;\ u = 2x^2+1$

$du = 4x\,dx$

$= \int_{1}^{3} \frac{1}{u^2}\,du$

$u(1) = 2(1)^2+1 = 3$

$u(0) = 2(0)^2+1 = 1$

$= \frac{-1}{u^1}\Big]_{1}^{3} = \frac{-1}{3} + 1 = \boxed{\frac{2}{3}}$

19) $\int_{-1}^{2} -\frac{4x}{(2x^2+1)^2}\,dx;\ u = 2x^2+1$

$du = 4x\,dx$

$u(2) = 2(2)^2+1 = 9$

$u(-1) = 2(-1)^2+1$

$-\int_3^9 \frac{1}{u^2}\,du$

$\left.\frac{-u^{-1}}{-1}\right]_3^9 = \frac{1}{9} - \frac{1}{3}$

$= \frac{1}{9} - \frac{3}{9} = \boxed{\frac{2}{9}}$

20) $\int_0^2 \frac{\cancel{12x}}{(2x^2+1)^2}\,dx;\ u = 2x^2+1$ $\quad 3\cdot 4$

$du = 4x\,dx$

$u(2) = 2(2)^2+1$

$u(0) = 2(0)^2+1$

$3\int_1^9 \frac{1}{u^2}\,du$

$= \left.\frac{3u^{-1}}{-1}\right]_1^9 = -3\left(\frac{1}{9} - 1\right)$

$= -3\left(\frac{1}{9} - \frac{9}{9}\right)$

$= -3\left(\frac{-8}{9}\right) = \boxed{\frac{8}{3}}$

$\frac{1}{u^2} = u^{-2}$

21) $\int_{-3}^{0} -\frac{\cancel{4x}}{(x^2+3)^2}\,dx$ $\quad 2\cdot 2$

$u = x^2+3$

$du = 2x\,dx$

$u(0) = 0^2+3$

$u(-3) = (-3)^2+3$

$-2\int_{12}^{3} \frac{1}{u^2}\,du$

$\left.\frac{-2u^{-1}}{-1}\right]_{12}^{3} = 2\left(\frac{1}{3} - \frac{1}{12}\right)$

$= 2\left(\frac{4}{12} - \frac{1}{12}\right)$

$= 2\left(\frac{3}{12}\right) = \boxed{\frac{1}{2}}$

22) $\int_{-2}^{0} \frac{\cancel{16x}}{(4x^2+2)^2}\,dx$ $\quad 2\cdot 8$

$u = 4x^2+2$

$du = 8x\,dx$

$u(0) = 4(0)^2+2$

$u(-2) = 4(-2)^2+2$

$2\int_{18}^{2} \frac{1}{u^2}\,du$

$\left.\frac{2u^{-1}}{-1}\right]_{18}^{2} = -2\left(\frac{1}{2} - \frac{1}{18}\right)$

$= -2\left(\frac{9}{18} - \frac{1}{18}\right)$

$= -2\left(\frac{8}{18}\right) = \boxed{\frac{-8}{9}}$

23) $\int_0^1 -\frac{4x}{(2x^2+2)^2}\,dx$

$u = 2x^2+2$

$du = 4x\,dx$

$u(1) = 2(1)^2+2$

$u(0) = 2(0)^2+2$

$-\int_2^4 \frac{1}{u^2}\,du$

$\left.\frac{-u^{-1}}{-1}\right]_2^4 = \left(\frac{1}{4} - \frac{1}{2}\right)$

$= \left(\frac{1}{4} - \frac{2}{4}\right)$

$= \boxed{-\frac{1}{4}}$

24) $\int_{-3}^{0} \frac{\cancel{8x}}{(2x^2+2)^2}\,dx$ $\quad 2\cdot 4$

$u = 2x^2+2$

$du = 4x\,dx$

$u(0) = 2(0)^2+2$

$u(-3) = 2(-3)^2+2$

$2\int_{20}^{2} \frac{1}{u^2}\,du$

$\left.\frac{2u^{-1}}{-1}\right]_{20}^{2} = -2\left(\frac{1}{2} - \frac{1}{20}\right)$

$= -2\left(\frac{10}{20} - \frac{1}{20}\right)$

$= -2\left(\frac{9}{20}\right) = \boxed{-\frac{9}{10}}$