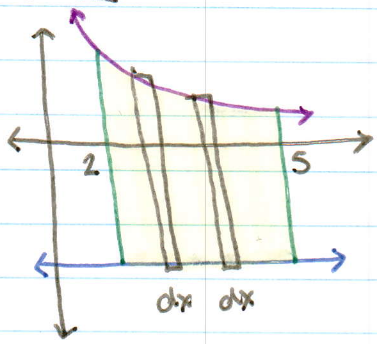
Areas in the Plane

Stew Dent
date Per

#1-16

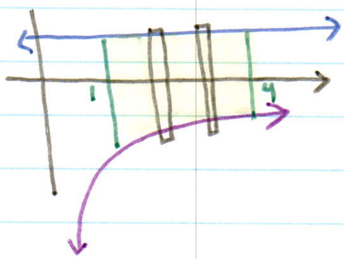
1) 

$y = 3/x^2$

$y = -4$

$x = 2, 5$

$$\int_2^5 \left(\frac{3}{x^2} - (-4)\right) dx$$

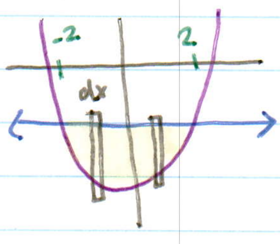
2) 

$y = -4/x^2$

$y = 1$

$x = 1, 4$

$$\int_1^4 \left(1 - \left(\frac{-4}{x^2}\right)\right) dx$$

3) 

$y = \frac{x^2}{2} - 6$

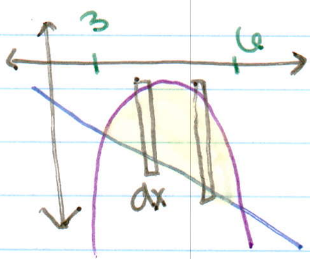
$y = -4$

$\frac{x^2}{2} - 6 = -4$

$\frac{x^2}{2} = 2$

$x^2 = 4 \rightarrow x = \pm 2$

$$\int_{-2}^{2} \left(-4 - \left(\frac{x^2}{2} - 6\right)\right) dx$$

4) 

$y = -x^2 + 8x - 18$

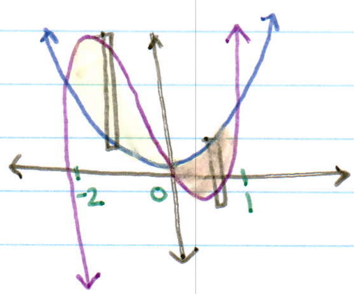
$y = -x$

$-x^2 + 8x - 18 = -x$

$0 = x^2 - 9x + 18$

$0 = (x-6)(x-3) \rightarrow x = 3, 6$

$$\int_3^6 \left(\left(-x^2 + 8x - 18\right) - (-x)\right) dx$$

5) 

$y = 2x^3 + 3x^2 - 4x$

$y = x^2$

$2x^3 + 3x^2 - 4x = x^2$

$2x^3 + 2x^2 - 4x = 0$

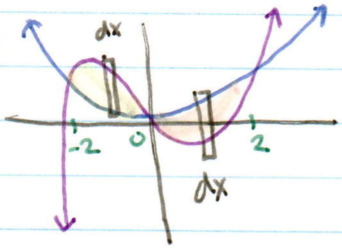
$2x(x^2 + x - 2) = 0$

$2x(x+2)(x-1) = 0 \rightarrow x = 0, 1, -2$

$$\int_{-2}^{0} \left(2x^3 + 3x^2 - 4x - x^2\right) dx$$

$$\int_0^1 x^2 - \left(2x^3 + 3x^2 - 4x\right) dx$$

6)



$$y = \frac{x^3}{2} + \frac{x^2}{2} - 2x$$

$$y = x^2/2$$

$$\frac{x^3}{2} + \frac{x^2}{2} - 2x = \frac{x^2}{2}$$

$$x^3 + x^2 - 4x = x^2$$

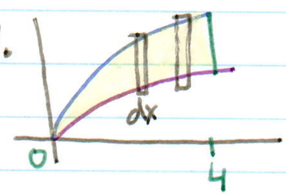
$$x^3 - 4x = 0$$

$$x(x+2)(x-2) = 0 \rightarrow x = 0, \pm 2$$

$$\int_{-2}^{0} \left(\frac{x^3}{2} + \frac{x^2}{2} - 2x - \frac{x^2}{2} \right) dx +$$

$$\int_{0}^{2} \left(\frac{x^2}{2} - \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x \right) \right) dx$$

7.



$$y = 2\sqrt{x}$$

$$y = 3\sqrt{x}$$

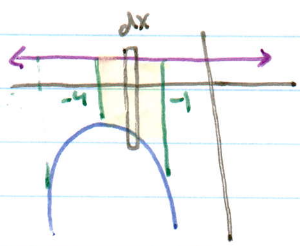
$$x = 0, 4$$

$$\int_0^4 (3\sqrt{x} - 2\sqrt{x})\, dx$$

$$= \int_0^4 \sqrt{x}\, dx = \frac{2 \cdot X^{3/2}}{3} \Big]_0^4$$

$$= \frac{2}{3}\sqrt{4}^{\,3} - \frac{2}{3}\sqrt{0}^{\,3} = \frac{2 \cdot 8}{3} - 0 = \boxed{\frac{16}{3}}$$

8.



$$y = -\frac{x^2}{2} - 4x - 10$$

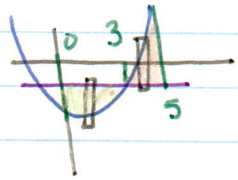
$$y = 2$$

$$x = -4, -1$$

$$\int_{-4}^{-1} 2 - \left(-\frac{x^2}{2} - 4x - 10 \right) dx = \int_{-4}^{-1} \frac{x^2}{2} + 4x + 12\, dx = \frac{x^3}{6} + 2x^2 + 12x \Big]_{-4}^{-1}$$

$$= \left(\frac{(-1)^3}{6} + 2(-1)^2 + 12(-1) \right) - \left(\frac{(-4)^3}{6} + 2(-4)^2 + 12(-4) \right)$$

$$= \left(-\frac{1}{6} + 2 - 12 \right) - \left(-\frac{32}{3} + 32 - 48 \right) = -\frac{1}{6} + \frac{64}{6} + 6 = \frac{63}{6} + \frac{36}{6} = \frac{99}{6} = \boxed{\frac{33}{2}}$$

9) 

$y = \frac{x^2}{2} - x - \frac{7}{2}$

$y = -2$

$x = 0, 5$

$\frac{x^2}{2} - x - \frac{7}{2} = -2$

$x^2 - 2x - 7 = -4$

$x^2 - 2x - 3 = 0 - (x-3)(x+1)$

$\rightsquigarrow x = 3$

$\frac{7}{2} - \frac{4}{2}$

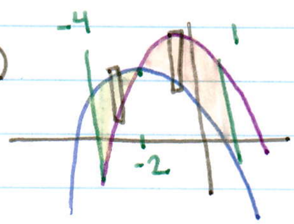
$$\int_0^3 -2 - \left(\frac{x^2}{2} - x - \frac{7}{2}\right) dx + \int_3^5 \frac{x^2}{2} - x - \frac{7}{2} - (-2)\, dx$$

$$= \int_0^3 -\frac{x^2}{2} + x + \frac{3}{2}\, dx + \int_3^5 \frac{x^2}{2} - x - \frac{3}{2}\, dx$$

$$= -\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{2}x \Big]_0^3 + \frac{x^3}{6} - \frac{x^2}{2} - \frac{3}{2}x \Big]_3^5$$

$$= \left(-\frac{3^3}{6} + \frac{3^2}{2} + \frac{3\cdot 3}{2}\right) - 0 + \left(\frac{5^3}{6} - \frac{5^2}{2} - \frac{3\cdot 5}{2}\right) - \left(\frac{3^3}{6} - \frac{3^2}{2} - \frac{3\cdot 3}{2}\right)$$

$$= \frac{9}{2} + \frac{5}{6} - (-9/2) = \boxed{59/6}$$

10) 

$y = -\frac{x^2}{2} - 2x + 1$

$y = -\frac{x^2}{2} + 5$

$x = -4, 1$

$-\frac{x^2}{2} - 2x + 1 = -\frac{x^2}{2} + 5$

$-2x + 1 = 5$

$-2x = 4$

$x = -2$

$$\int_{-4}^{-2} -\frac{x^2}{2} - 2x + 1 - \left(-\frac{x^2}{2} + 5\right) dx + \int_{-2}^{1} -\frac{x^2}{2} + 5 - \left(-\frac{x^2}{2} - 2x + 1\right) dx$$

$$= \int_{-4}^{-2} -2x - 4\, dx + \int_{-2}^{1} 2x + 4\, dx$$

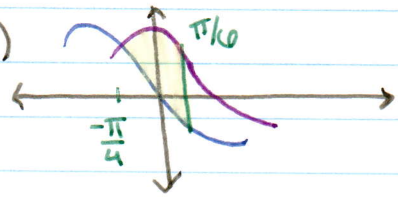
$$= -x^2 - 4x \Big]_{-4}^{-2} + x^2 + 4x \Big]_{-2}^{1}$$

$$= \left(-(-2)^2 - 4(-2)\right) - \left(-(-4)^2 - 4(-4)\right) + \left(1^2 + 4(1)\right) - \left((-2)^2 + 4(-2)\right)$$

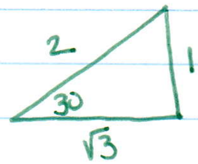
$$= 4 - 0 + 5 - (-4)$$

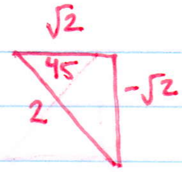
$$= 13$$

11)



$y = -2\sin x$
$y = 2\cos x$
$x = -\pi/4, \pi/6$





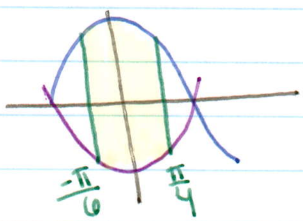
$$\int_{-\pi/4}^{\pi/6} 2\cos x - {}^{-}2\sin x \, dx = \int_{-\pi/4}^{\pi/6} 2\cos x + 2\sin x \, dx$$

$$= 2\sin x - 2\cos x \Big]_{-\pi/4}^{\pi/6} = 2\sin\frac{\pi}{6} - 2\cos\frac{\pi}{6} - \left(2\sin\frac{-\pi}{4} - 2\cos\frac{-\pi}{4}\right)$$

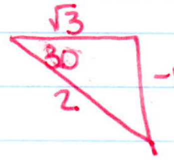
$$= 2\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right) - \left(2\left(\frac{-\sqrt{2}}{2}\right) - 2\left(\frac{\sqrt{2}}{2}\right)\right)$$

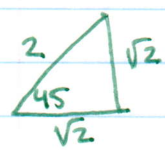
$$= 1 - \sqrt{3} + \sqrt{2} + \sqrt{2} = \boxed{2\sqrt{2} + 1 - 3}$$

12)



$y = 2\cos x$
$y = -2\cos x$
$x = \frac{-\pi}{6}, \pi/4$





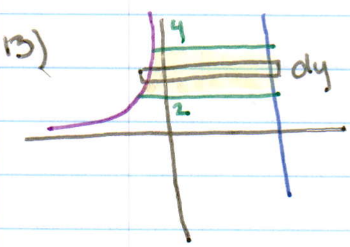
$$\int_{-\pi/6}^{\pi/4} 2\cos x - {}^{-}2\cos x \, dx = \int_{-\pi/6}^{\pi/4} 4\cos x \, dx = 4\sin x \Big]_{-\pi/6}^{\pi/4}$$

$$= 4\left(\sin\frac{\pi}{4} - \sin\frac{-\pi}{6}\right)$$

$$= 4\left(\frac{\sqrt{2}}{2} - \frac{-1}{2}\right)$$

$$= \boxed{2\sqrt{2} + 2}$$

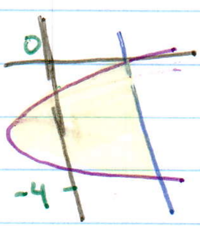
13)



$x = \frac{-1}{y^2}$
$x = 3$
$y = 2, 4$

$$\int_2^4 \left(3 - \frac{-1}{y^2}\right) dy$$

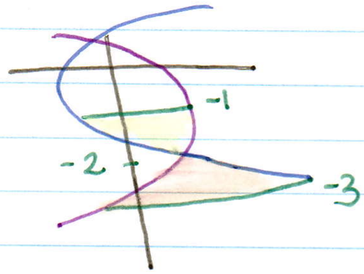
14)



$x = y^2 + 4y + 3$
$x = 3$
$y = 0, -4$

$$\int_0^{-4} 3 - (y^2 + 4y + 3) \, dy$$

15)



$x = y^2 - 2$

$x = -y^2 - 2y + 2$

$y = 1, -3$

$y^2 - 2 = -y^2 - 2y + 2$

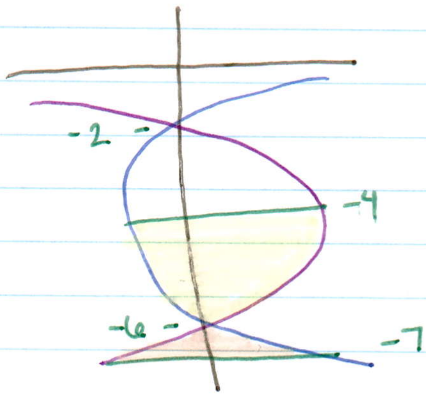
$2y^2 + 2y - 4 = 0$

$2(y^2 + y - 2) = 0$

$2(y+2)(y-1) = 0$

$$\int_{-3}^{-2} (y^2-2) - (-y^2-2y+2)\,dy + \int_{-2}^{-1} (-y^2-2y+2) - (y^2-2)\,dy$$

16)



$x = -y^2 - 8y - 12$

$x = y^2 + 8y + 12$

$y = -4, -7$

$y^2 + 8y + 12 = -y^2 - 8y - 12$

$2y^2 + 16y + 24 = 0$

$2(y^2 + 8y + 12) = 0$

$2(y+6)(y+2) = 0$

$y = -6, -2$

$$\int_{-7}^{-6} y^2 + 8y + 12 - (-y^2 - 8y - 12)\,dy$$

$$+ \int_{-6}^{-4} -y^2 - 8y - 12 - (y^2 + 8y + 12)\,dy$$