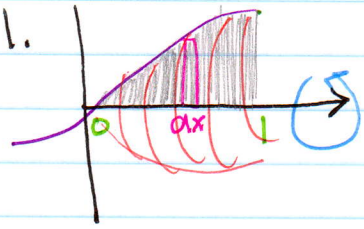
Volume: Rotations and the Disk Method
#1-12

Stew Dent
date Per

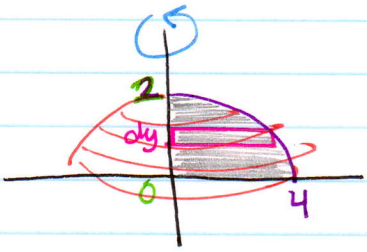
1.



radius = ax - fn
$y = 0 - \sqrt[3]{x}$
$y = -\sqrt[3]{x}$

disc
$\pi \int_0^1 (0 - \sqrt[3]{x})^2 dx$

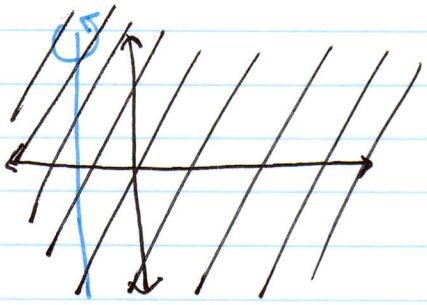
2.



radius = ax - fn
$x = 0 - (-y^2+4)$
$x = -(-y^2+4)$

disk $\pi \int_0^2 (0-(-y^2+4))^2 dy$

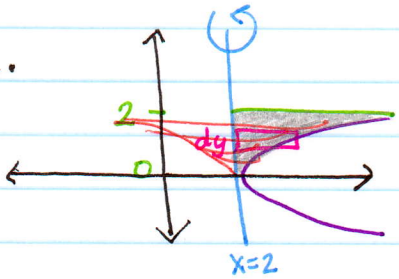
3.



radius = ax - fn
$y = -1 - (x^2-1)$

disk → $\pi \int_0^2 (-1-(x^2-1))^2 dx$

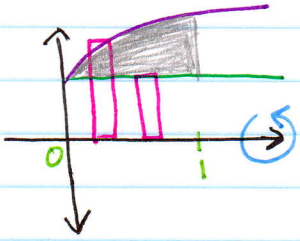
4.



radius = ax - fn
$x = 2 - (y^2+2)$
$x = -(y^2+2)$

disk → $\pi \int_0^2 (2-(y^2+2))^2 dy$

5.

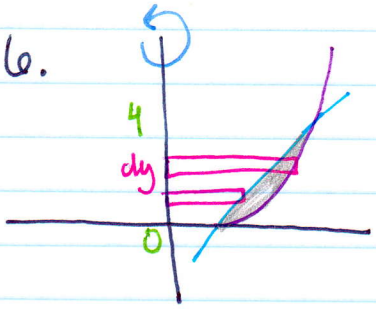


Radius = ax - fn
$y = 0 - (\sqrt{x}+2)$
$y = -(\sqrt{x}+2)$

radius = ax - fn
$y = 0 - 2$
$y = -2$

Washer
$\pi \int_0^1 (0-(\sqrt{x}+2))^2 - (0-2)^2 dx$

6.



$\sqrt{y}+1=\frac{y}{2}+1$

$\sqrt{y}=y/2$

$y=y^2/4$

$4y=y^2$

$y^2-4y=0$

$y(y-4)=0$

$y=0,4$

Radius = ax - fn

$x=0-(\sqrt{y}+1)$

$x=$

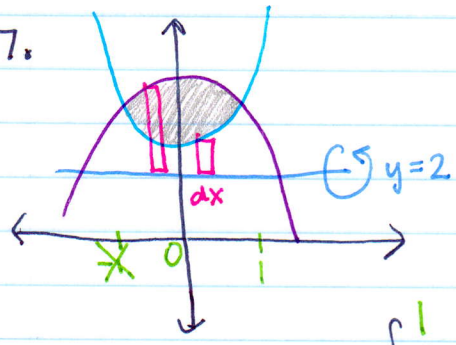
radius = ax - fn

$x=0-(\frac{y}{2}+1)$

$x=$

Washer → $\pi\int_0^4 (0-(\sqrt{y}+1))^2 - (0-(\frac{y}{2}+1))^2\,dy$

7.



$-x^2+5=x^2+3$

$2=2x^2$

$1=x^2$

$x=\pm 1$

Radius = ax - fn

$y=2-(-x^2+5)$

$y=$

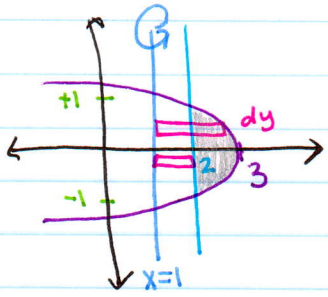
radius = ax - fn

$y=2-(x^2+3)$

$y=$

Washer → $\pi\int_0^1 (2-(-x^2+5))^2 - (2-(x^2+3))^2\,dx$

8.



$-y^2+3=2$

$y^2=1$

$y=\pm 1$

Radius = ax - fn

$x=1-(-y^2+3)$

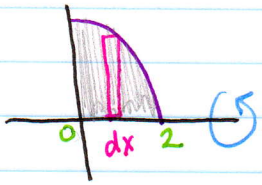
$x=$

radius = ax - fn

$x=1-2$

$x=$

Washer → $\pi\int_{-1}^1 (1-(-y^2+3))^2 - (1-2)^2\,dy$

9. 

Radius = ax - fn

$y = 0 \quad y = -(-x^2+4)$

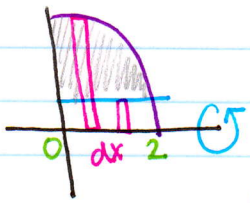
disk → $\pi \int_0^2 (0-(-x^2+4))^2 \, dx = \pi \int_0^2 (x^2-4)^2 \, dx$

$(x^2+4)(x^2-4)$

$= \pi \int_0^2 (x^4 - 8x^2 + 16) \, dx = \pi \left(\frac{x^5}{5} - \frac{8x^3}{3} + 16x \right) \Big]_0^2$

$= \pi \left[\left(\frac{2^5}{5} - \frac{8 \cdot 2^3}{3} + 16(2) \right) - \left(\frac{0^5}{5} - \frac{8 \cdot 0^3}{3} + 16(0) \right) \right]$

$= \pi \left(\frac{32}{5} - \frac{64}{3} + 32 \right) = \pi \left(\frac{96}{15} - \frac{320}{15} + \frac{480}{15} \right) = \boxed{\frac{256\pi}{15}}$

10. 

Radius = $0 - (-x^2+5)$

radius = $0 - 1$

~~Washer~~ → $\pi \int_0^2 (0-(-x^2+5))^2 - (0-1)^2 \, dx$

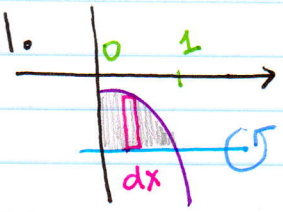
$(x^2-5)(x^2-5)$

$= \pi \int_0^2 x^4 - 10x^2 + 25 - 1 \, dx$

$= \pi \left(\frac{x^5}{5} - \frac{10x^3}{3} + 24x \right) \Big]_0^2$

$= \pi \left[\left(\frac{2^5}{5} - \frac{10 \cdot 2^3}{3} + 24(2) \right) - \left(\frac{0^5}{5} - \frac{10 \cdot 0^3}{3} + 24(0) \right) \right]$

$= \pi \left[\frac{32}{5} - \frac{80}{3} + 48 \right] = \pi \left[\frac{96}{15} - \frac{400}{15} + \frac{720}{15} \right] = \boxed{\frac{416\pi}{15}}$

11. 

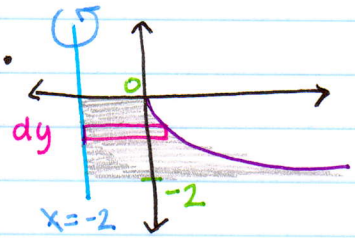
Radius = $-2 - (-x^2-1)$
($y=$ $y=$)

$$\text{disk} \rightarrow \pi \int_0^1 (-2-(-x^2-1))^2 dx \qquad -2+x^2+1 = x^2-1$$

$$(x^2-1)(x^2-1)$$

$$= \pi \int_0^1 x^4 - 2x^2 + 1 \, dx$$

$$= \pi \left(\frac{x^5}{5} - \frac{2x^3}{3} + x \right) \Big]_0^1 = \pi \left[\left(\frac{1^5}{5} - \frac{2 \cdot 1^3}{3} + 1 \right) - 0 \right] = \pi \left[\frac{3}{15} - \frac{10}{15} + \frac{15}{15} \right] = \boxed{\frac{8\pi}{15}}$$

12. 

Radius = ax - fn
$-2 - (y^2)$
($x=$ $x=$)

$$\text{disk} \rightarrow \pi \int_{-2}^0 (-2-y^2)^2 dy \qquad (-2-y^2)(-2-y^2)$$

$$= \pi \int_{-2}^0 y^4 + 4y^2 + 4 \, dy = \pi \left(\frac{y^5}{5} + \frac{4y^3}{3} + 4y \right) \Big]_{-2}^0$$

$$= \pi \left(\left(\frac{0^5}{5} + \frac{4 \cdot 0^3}{3} + 4 \cdot 0 \right) - \left(\frac{(-2)^5}{5} + \frac{4(-2)^3}{3} + 4(-2) \right) \right]$$

$$= \pi \left(- \left(\frac{-32}{5} + \frac{-32}{3} - 8 \right) \right) = -\pi \left(\frac{-96}{15} + \frac{-160}{15} - \frac{120}{15} \right) = \cancel{\text{XXXXXX}} =$$

$$= -\pi \left(\frac{-376}{15} \right)$$

$$= \frac{376\pi}{15}$$