

# Asymptotes: Limits Involving Infinity

#1 = 40

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Date Per

1.  $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = \frac{1}{0}$  undef  
 $\Rightarrow \frac{1}{2^+-2} = \frac{+}{+} \rightarrow +\infty$

2.  $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$   
 $\frac{1}{2^- - 2} = \frac{+}{-}$

3.  $\lim_{x \rightarrow 2} \frac{1}{x-2} = \text{DNE}$

4.  $\lim_{x \rightarrow 3^-} \frac{x}{x-3} = \frac{3}{0}$  undef  
 $\rightarrow \frac{3}{3^- - 3} = \frac{+}{-} \rightarrow -\infty$

5.  $\lim_{x \rightarrow 3^+} \frac{x}{x-3} = +\infty$   
 $\frac{3}{3^+ - 3} = \frac{+}{+}$

6.  $\lim_{x \rightarrow 3} \frac{x}{x-3} = \text{DNE}$

7.  $\lim_{x \rightarrow 3^+} \frac{1}{(x-3)^2} = \frac{1}{0}$  undef  
 $\rightarrow \frac{1}{(3^+ - 3)^2} = \frac{+}{+} \rightarrow +\infty$

8.  $\lim_{x \rightarrow 3^-} \frac{1}{(x-3)^2} = +\infty$   
 $\frac{1}{(3^- - 3)^2} = \frac{+}{+}$

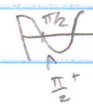
9.  $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = +\infty$

10.  $\lim_{y \rightarrow 6^+} \frac{y+6}{y^2-36} = \frac{0}{0}$   
 $\frac{y+6}{(y+6)(y-6)}$   
 $\lim_{y \rightarrow 6^+} \frac{1}{y-6} = \frac{1}{0}$  undef  
 $\frac{1}{6^+ - 6} = \frac{+}{+} \rightarrow +\infty$

11.  $\lim_{x \rightarrow 0^+} \csc x = \lim_{x \rightarrow 0^+} \frac{1}{\sin x} = \frac{1}{0}$  undef  
 $\frac{1}{\sin 0^+} = \frac{1}{+} = +\infty$

12.  $\lim_{x \rightarrow 0^-} \csc x = -\infty$   
 $\frac{1}{\sin 0^-} = \frac{1}{-}$

13.  $\lim_{x \rightarrow \pi/2^+} \sec x = \lim_{x \rightarrow \pi/2^+} \frac{1}{\cos x} = \frac{1}{0}$  undef  
 $\frac{1}{\cos \pi/2^+} = \frac{1}{-} \rightarrow -\infty$



14.  $\lim_{x \rightarrow 4} \frac{3-x}{x^2-2x-8} = \frac{0}{0}$  undef  
 $\lim_{x \rightarrow 4} \frac{(3-x)}{(x-4)(x+2)}$

15.  $\lim_{x \rightarrow 5^+} \frac{x-5}{|x-5|} \rightarrow \frac{5^+ - 5}{+} = \frac{+}{+}$   
 $\frac{x-5}{+} = \frac{+}{+}$

$x \rightarrow 4^+ \quad \frac{+}{(+)(+)} \quad f(x) \rightarrow +\infty$

$x \rightarrow 4^- \quad \frac{+}{(-)(+)} \quad f(x) \rightarrow -\infty$

$\lim_{x \rightarrow 4} \frac{3-x}{x^2-2x-8} = \text{DNE}$

$\lim_{x \rightarrow 5^+} \frac{x-5}{|x-5|} = +1$

16.  $\lim_{x \rightarrow -3} \frac{|x+3|}{x+3} = \text{DNE}$   
 $\frac{|x+3|}{x+3} = \frac{+}{+}$

17.  $\lim_{x \rightarrow 3} \frac{-x^2}{x^2-6x+9} = \frac{3}{0}$  undef  
 $\lim_{x \rightarrow 3} \frac{-x^2}{(x-3)^2}$

$x \rightarrow -3^+ \quad \frac{+}{+} \rightarrow +1$

$x \rightarrow -3^- \quad \frac{+}{-} \rightarrow -1$

$x \rightarrow 3^- \quad \frac{-}{+}$   
 $x \rightarrow 3^+ \quad \frac{-}{+} \quad \left. \vphantom{\begin{matrix} x \rightarrow 3^- \\ x \rightarrow 3^+ \end{matrix}} \right\} \rightarrow -\infty$

18.  $\lim_{x \rightarrow \infty} 6 = 6$

19.  $\lim_{x \rightarrow -\infty} (3x^4 - 6x^3 + 5x^2 - 145x + 12) = \infty$



$$20. \lim_{x \rightarrow \infty} \frac{2x-3}{4x+5} = \frac{1}{2}$$

$$21. \lim_{x \rightarrow \infty} \frac{x}{|x|} = 1$$

$$22. \lim_{x \rightarrow -\infty} \frac{x}{|x|} = -1$$

$$\frac{-\infty}{|-\infty|} = -1$$

$$23. \lim_{x \rightarrow -\infty} \frac{2x-1}{|x|-3} = -2$$

$$\frac{2(-\infty)}{|-\infty|} = -2$$

$$24. \lim_{x \rightarrow \infty} \frac{2x-1}{|x|-3} = +2$$

$$\frac{2(\infty)}{|\infty|} = +2$$

$$25. \lim_{x \rightarrow \infty} \frac{7-3x^3}{2x^3+1} = -\frac{3}{2}$$

$$26. \lim_{x \rightarrow -\infty} \frac{7-3x^3}{2x^3+1} = -\frac{3}{2}$$

$$27. \lim_{x \rightarrow -\infty} \frac{4x^2-3x-2-5x^3}{x^2+x+7} = \infty$$

$$\frac{-5x^3}{0x^3} = \text{Undef} = \pm \infty$$

$$\frac{-5(-\infty)^3}{0}$$

$$28. \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+4}} = +1$$

$$\frac{\infty}{\sqrt{\infty^2}}$$

$$29. \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+4}} = -1$$

$$\frac{-\infty}{\sqrt{(-\infty)^2}}$$

$$30. \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+x}}{x^2+1} = 0$$

$$31. \lim_{x \rightarrow \infty} \frac{\ln x^2}{\ln x} = \lim_{x \rightarrow \infty} \frac{2 \cdot \ln x}{\ln x} = \lim_{x \rightarrow \infty} 2 = 2$$

$$\sqrt{3x^2+x} \frac{0x^2}{x^2} =$$

$$32. \lim_{x \rightarrow \infty} \frac{\ln x}{\log x} = \lim_{x \rightarrow \infty} \frac{1}{\log e} = \frac{1}{\log e}$$

$$\frac{\ln x}{\log x} = \frac{\log_e x}{\log_{10} x} = \frac{\log_{10} x}{\log_{10} e} \cdot \frac{1}{\log_{10} x} = \frac{1}{\log_e}$$

$$33. \lim_{x \rightarrow \infty} \frac{e^x}{\ln x} = \infty$$

$e^x > \ln x$   
for large  $x$  top heavy

$$34. \lim_{x \rightarrow \infty} \frac{x^e}{e^x} = 0$$

$$x^e < e^x$$

for large  $x$  bottom heavy

35. Horizontal asymptote :  $x \rightarrow \pm\infty$   $f(x) \rightarrow \#$   
Vertical asymptote :  $x \rightarrow a$   $f(x) \rightarrow \pm\infty$   
↳ (undefined when plugin  $x=a$ )

36. **False**. if  $(x-1)$  is a factor of  $P(x)$ , then you have a POINT discontinuity

Counter ex  $f(x) = \frac{(x+1)(x+2)}{\cancel{(x+1)}}$

37. **False**, If every factor in the denominator reduces  $f(x) = \frac{x+1}{x+1}$   
or if the denominator  $\neq 0$   $f(x) = \frac{3}{x^2+1}$

38. **True**, Polynomials are cts functions

39. **False**, you could have a hole

$$f(x) = \frac{(x-a)(x+1)}{\cancel{(x-a)}}$$

$f(a) = \text{undefined}$  but not an asymptote

40. **True**, at most fn's can have TWO H.A.  
 $x \rightarrow +\infty$  and  $x \rightarrow -\infty$