5 Limits and Continuity
#1-15

StewDent
date Per

1. $\lim_{x\to 3^-} 2x-3 = 2(3)-3 = 3$ $\quad \lim_{x\to 3} f(x) = \text{DNE}$

$\lim_{x\to 3^+} x-1 = 3-1 = 2$

2. $\lim_{x\to -1^-} 2x = 2(-1) = -2$ $\quad \lim_{x\to 3} f(x) = -2$

$\lim_{x\to -1^+} x^3-1 = (-1)^3-1 = -2$

3. $\lim_{x\to 1^-} \frac{x}{x-1} = \frac{1}{1-1} = \frac{1}{0} \to \pm\infty$ more work

$= \frac{1}{1^--1} = \frac{1}{-\#} = -\infty$ $\quad \lim_{x\to 1} f(x) = -\infty$

$\lim_{x\to 1^+} \frac{x-2}{x-1} = \frac{-1}{0} \to \pm\infty$

$= \frac{-1}{1^+-1} = \frac{-\#}{+} = -\infty$

4. $\lim_{x\to 2} f(x) = f(2)$

$\lim_{x\to 2} x^2-2x-3 = k-3$

$(2)^2-2(2)-3 = k-3$

$4-4-3 = k-3$

$-3 = k-3$

$k=0$

5. $\lim_{x\to 7} f(x) = f(7)$

$\lim_{x\to 7} \frac{x^2-49}{x-7} = k^2-2$

$\lim_{x\to 7} (x+7) = k^2-2$

$14 = k^2-2$

$16 = k^2$

$\boxed{k=\pm 4}$

6. a) Jump discont. @ $x=c$ $\quad \lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$

b) Point disc. @ $x=c$ $\quad f(c)$ does not exist

c) " " $\quad \lim_{x\to c} f(x) \neq f(c)$

d) same as a

7. A non removeable discontinuity is either a jump disc.

(a) $f(x) = \begin{cases} x^2 & x<2 \\ x^2+4 & x \geq 2 \end{cases}$ $\quad \lim_{x\to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

or a vertical asymptote

$f(x) = \dfrac{1}{x-2}$ $\quad \lim_{x\to 2} f(x) = \pm\infty$, DNE

A removeable discontinuity is a point discontinuity

so $\lim_{x\to 2} f(x)$ exists ($\frac{0}{0} \to$ more work)

(b) $\longrightarrow$ $f(x) = \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

(c) $\rightarrow$ $f(x) = \dfrac{(x-2)}{\underset{\uparrow}{(x-2)}\ \underset{\uparrow}{(x+2)}}$

removeable nonremoveable

8. a) f is cts at $x=4$

b) f is cts at $x=-2$

c) f has a point disc at $x=3$

d) f has a point disc at $x=-3$

e) f has a jump disc at $x=2$

f) f has a jump disc at $x=-1$

9. ① $\lim_{x\to 1^-} x = 1$

$\lim_{x\to 1^+} x^2 = 1$

$\lim_{x\to 1} f(x) = 1$ ✓

② $f(1) = 1^2$

$f(1) = 1$ ✓

③ $\lim_{x\to 1} f(x) = f(1)$

$1 = 1$ ✓

10. ① $\lim_{x\to 1^-} -2(1)+3 = 1$

$\lim_{x\to 1^+} x^2 = 1$

$\lim_{x\to 1} f(x) = 1$ ✓

② $f(1) = 1^2$

$f(1) = 1$

$\lim_{x\to 1} f(x) = f(1)$

$1 = 1$ ✓

11. f is cts thus $\lim_{x\to 3} f(x) = f(3)$

since $\lim_{x\to 3} f(x) = 1/2$ $\quad \boxed{f(3) = \frac{1}{2}}$

12. want $e^{-x} = x \rightarrow 0 = e^{-x} - x$

$f(x) = e^{-x} - x$ want to show $f(c) = 0$

$f(0) = e^0 - 0$
$= 1 - 0$
$= 1$

$f(1) = \frac{1}{e} - 1$
< 0 ← $1 > \frac{1}{e}$

Because e^{-x} and x are both cts, $f(x) = e^{-x} - x$ is also cts, and $f(1) < 0 < f(0)$ by the Intermediate Value theorem the exists a value c in $[0,1]$ such that $f(c) = 0$.

13. $f(2) = 2^2 - 4(2) + 3 = -1$
$f(4) = 4^2 - 4(4) + 3 = 3$

f is a polynomial so f is cts, also $f(2) < 0 < f(4)$ thus by the IVT there exists a c in $[2,4]$ such that $f(c) = 0$.

14. Since $f(x) = \frac{1}{x}$ is <u>not</u> cts on $[-1,1]$ we cannot determine if $f(x)$ has a zero on $[-1,1]$

15. f has vertical asymptotes at $x = \pm 2$ ($x^2 - 4 = 0$, $x^2 = 4$, $x = \pm 2$)
however on the interval $[-1,1]$ f is cts.

$f(-1) = \frac{1}{(-1)^2 - 4} = -\frac{1}{3}$
$f(1) = \frac{1}{1^2 - 4} = -1/3$

However 0 is not <u>between</u> $f(-1)$ and $f(1)$ so we cannot guarantee $f(x)$ has a zero in $[-1,1]$.