6 Definition of the Derivative & Differentiability
1-10

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1. $f(x+h) = (x+h)$

$f(x) = x$

$\boxed{f'(x) = 1}$

$$f'(x) = \lim_{h\to 0} \frac{(x+h) - (x)}{h}$$
$$= \lim_{h\to 0} \frac{h}{h}$$
$$= \lim_{h\to 0} 1 = 1$$

2. $f(x+h) = (x+h)^2$

$= x^2 + 2xh + h^2$

$f(x) = x^2$

$\boxed{f'(x) = 2x}$

$$f'(x) = \lim_{h\to 0} \frac{x^2+2xh+h^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2}{h}$$
$$= \lim_{h\to 0} 2x+h$$
$$= 2x$$

3. $f(x+h) = (x+h)^2 + 3(x+h)$

$= x^2 + 2xh + h^2 + 3x + 3h$

$f(x) = x^2 + 3x$

$\boxed{f'(x) = 2x + 3}$

$$f'(x) = \lim_{h\to 0} \frac{x^2+2xh+h^2+3x+3h - x^2 - 3x}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2+3h}{h}$$
$$= \lim_{h\to 0} 2x + h + 3$$
$$= 2x + 3$$

4. $f(x+h) = 3$

$f(x) = 3$

$\boxed{f'(x) = 0}$

$$f'(x) = \lim_{h\to 0} \frac{3-3}{h}$$
$$= \lim_{h\to 0} \frac{0}{h}$$
$$= 0$$

5. $f(x+h) = 3(x+h)^2$

$= 3x^2 + 6xh + 3h^2$

$f(x) = 3x^2$

$\boxed{f'(x) = 6x}$

$$f'(x) = \lim_{h\to 0} \frac{3x^2+6xh+3h^2 - 3x^2}{h}$$
$$= \lim_{h\to 0} \frac{6xh+3h^2}{h}$$
$$= \lim_{h\to 0} 6x + 3h$$
$$= 6x$$

6. $f(x+h) = 4(x+h)^2 + 5(x+h) - 1$

$= 4x^2 + 8xh + 4h^2 + 5x + 5h - 1$

$f(x) = 4x^2 + 5x - 1$

$\boxed{f'(x) = 8x + 5}$

$f'(x) = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + 5x + 5h - 1 - 4x^2 - 5x + 1}{h}$

$= \lim_{h \to 0} \frac{8xh + 4h^2 + 5h}{h}$

$= \lim_{h \to 0} 8x + 4h + 5$

$= 8x + 5$

7. $x = -4$ has a cusp; $x = 0$ has a discontinuity

8. $f(x)$ has a vertical tangent line at $x = 3$

9. $x = -1$ is not cts, $x = 2$ has a cusp

10. $x = -1$ is not cts, $x = 0$ is not cts, $x = 1$ is a cusp, $x = 2$ is not cts, $x = 3$ is not cts