8 The Chain Rule

#1-15

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date per

1. $f(x) = (3x^2-14)^5$

$f'(x) = 5(3x^2-14)^4(6x)$

2. $f(x) = \sin(3x+1)$

$f'(x) = \cos(3x+1)\cdot(3)$

3. $y = (x+\sqrt{x})^{-2}$

$y' = -2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

4. $y = \sin^{-5}(x) + \cos^3(x)$

$y' = -5(\sin(x))^{-6} + 3\cos^2(x)$

5. $y = (1+\cos^2(7x))^3$

$y' = 3(1+\cos^2(7x))^2\cdot(2\cos(7x)\cdot\sin(7x)\cdot 7)$

6. $g(x) = \sqrt{\tan(5x)}$

$= (\tan(5x))^{1/2}$

$g'(x) = \frac{1}{2}(\tan(5x))^{-1/2}\cdot(\sec^2(5x))\cdot(5)$

7. $g(x) = \cos^2(x^3+x^2)$

$g'(x) = 2\cos(x^3+x^2)\cdot[-\sin(x^3+x^2)](3x^2+2x)$

8. $f(x) = \sqrt{3x^2-2}$ $\qquad (3,5) \qquad m = 9/5$

$f(3) = \sqrt{3(3)^2-2} = 5$

$f'(x) = \frac{1}{2}(3x^2-2)^{-1/2}(6x)$

$f'(3) = \dfrac{3\cancel{6}(3)}{\cancel{2}\sqrt{3(3)^2-2}} = \dfrac{9}{5}$

$\boxed{y-5 = \frac{9}{5}(x-3)}$

9. $y = \sin(2x)$ $\qquad (\pi,0) \qquad m=2$

$y(\pi) = \sin(2\pi) = 0$

$y' = \cos(2x)\cdot 2$

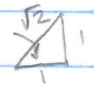
$y'(\pi) = 2\cdot\cos(2\pi) = 2\cdot 1 = 2$

$\boxed{y-0 = 2(x-\pi)}$

10. $f(x) = \tan^2(x)$ $\quad (\pi/4, 1) \quad m = 1$

$f(\pi/4) = \tan^2(\pi/4) = 1^2 = 1$

$f'(x) = 2\tan x \cdot \sec^2 x$

$45° = \pi/4$ 

$f'(\pi/4) = 2\tan(\pi/4) \cdot \sec^2(\pi/4)$

$= 2(1) \cdot \left(\frac{1}{\sqrt{2}}\right)^2 = 2 \cdot 1 \cdot \left(\frac{1}{2}\right) = 1$

$\boxed{y - 1 = 1(x - \pi/4)}$

11. $f(x) = 2(x^2-1)^3$

$f'(x) = 2 \cdot 3(x^2-1)^2 \cdot (2x) = (12x)(x^2-1)^2$

ok to stop here → $\boxed{f''(x) = 12(x^2-1)^2 + 2(x^2-1)(2x)(12x)}$

$= 12(x^2-1)^2 + 48x(x^2-1)$

$= 12(x^2-1)[x^2 - 1 + 4x]$

$12x \quad (x^2-1)^2$

$12 \quad 2(x^2-1)(2x)$

12. $g(x) = \frac{1}{x-2} = (x-2)^{-1}$

$g'(x) = -1(x-2)^{-2}$

$\boxed{g''(x) = (-1)(-2)(x-2)^{-3}}$

$= 2(x-2)^{-3}$

$= \frac{2}{(x-2)^3}$

13. $s(t) = \sin(t^2)$

$s'(t) = \cos(t^2) \cdot (2t)$

$\boxed{s''(t) = -\sin(t^2) \cdot 2t \cdot 2t + 2\cos(t^2)}$

$= -4t^2 \sin(t^2) + 2\cos(t^2)$

$\cos(t^2) \quad 2t$

$-\sin(t^2) \cdot 2t \quad 2$

14) A) $g(x) = f(x) - 2$

$\boxed{g'(x) = f'(x)}$

B) $h(x) = 2f(x)$

$\boxed{h'(x) = 2 \cdot f'(x)}$

C) $r(x) = f(-3x)$

$\boxed{r'(x) = f'(-3x) \cdot (-3)}$

D) $s(x) = f(x+2)$

$\boxed{s'(x) = f'(x+2) \cdot 1}$

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14 cont

x	-2	-1	0	1	2	3
$f'(x)$	4	$2/3$	$-1/3$	-1	-2	-4
$f'(x) = g'(x)$	4	$2/3$	$-1/3$	-1	-2	-4
$2\cdot f'(x) = h'(x)$	$2\cdot 4 = 8$	$\frac{2}{3}\cdot 2 = 4/3$	$-\frac{1}{3}\cdot 2 = \frac{-2}{3}$	$-1\cdot 2 = -2$	$-2\cdot 2 = -4$	$-4\cdot 2 = -8$
$-3\cdot f(-3x) = r'(x)$	$-3\cdot f'(6)$ =notposs	$-3\cdot f'(3)$ $= -3\cdot -4$ $= 12$	$-3\cdot f'(0)$ $-3\cdot -\frac{1}{3}$ $= 1$	$-3\cdot f'(-3)$ =notposs	$-3\cdot f'(-6)$ =notposs	$-3\cdot f'(-9)$ =notposs
$f'(x+2) = s'(x)$	$f'(0)$ $= -1/3$	$f'(1)$ $= -1$	$f'(2)$ $= -2$	$f'(3)$ $= -4$	$f'(4)$ =notposs	$f'(5)$ =notposs

15. A) $\frac{d}{dx}\left[5\cdot f(x) - g(x)\right]\Big|_{x=1} = 5\cdot f'(x) - g(x)\Big|_{x=1}$

$= 5\cdot f'(1) - g'(1)$

$= 5\left(-\frac{1}{3}\right) - \left(-\frac{8}{3}\right) = -\frac{5}{3} + \frac{8}{3} = \frac{3}{3} = \boxed{1}$

B) $\frac{d}{dx}\left[f(x)\cdot g^3(x)\right]\Big|_{x=0}$

$f \quad g^3$

$f' \quad 3g^2\cdot g'$

$= f'(x)g^3(x) + 3\cdot g^2(x)g'(x)f(x)\Big|_{x=0}$

$= f'(0)[g(0)]^3 + 3[g(0)]^2 g'(0)\cdot f(0)$

$= (5)(1)^3 + 3(1)^2\cdot\left(\frac{1}{3}\right)\cdot(1)$

$= 5 + 1 = \boxed{6}$

C) $\frac{d}{dx}\left[\frac{f(x)}{g(x)+1}\right]\Big|_{x=0}$

$f(x) \quad g(x)+1$

$f'(x) \quad g'(x)$

$= \frac{f'(x)(g(x)+1) - g'(x)f(x)}{(g(x)+1)^2}\Bigg|_{x=0}$

$= \frac{f'(0)(g(0)+1) - g'(0)f(0)}{(g(0)+1)^2}$

$= \frac{(5)(1+1) - \left(\frac{1}{3}\right)(1)}{(1+1)^2} = \frac{5(2) - \frac{1}{3}}{2^2} = \frac{29}{3} \div 4 = \boxed{\frac{29}{12}}$

D) $\frac{d}{dx}\left[f(g(x))\right]\Big|_{x=0} = f'(g(x)) \cdot g'(x)\Big|_{x=0}$

$$= f'(g(0)) \cdot g'(0)$$
$$= f'(1)\, g'(0)$$
$$= \left(-\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) = \boxed{-1/9}$$

E) $\frac{d}{dx}\left[g(f(x))\right]\Big|_{x=0} = g'(f(x)) \cdot f'(x)\Big|_{x=0}$

$$= g'(f(0)) \cdot f'(0)$$
$$= g'(1) \cdot f'(0)$$
$$= -\tfrac{8}{3} \cdot 5 = \boxed{-40/3}$$

F) $\frac{d}{dx}\left[(g(x)+f(x))^{-2}\right]\Big|_{x=1} = -2\left(g(x)+f(x)\right)^{-3} \cdot \left(g'(x)+f'(x)\right)\Big|_{x=1}$

$$= \frac{-2\left(g'(1)+f'(1)\right)}{(g(1)+f(1))^3}$$
$$= \frac{-2\left(-\frac{8}{3} + -\frac{1}{3}\right)}{(-4+3)^3} = \frac{-2\left(-9/3\right)}{(-1)^3} = \frac{-2(-3)}{-1}$$
$$= \boxed{-6}$$

G) $\frac{d}{dx}\left[f(x+g(x))\right]\Big|_{x=0} = f'(x+g(x)) \cdot (1+g'(x))\Big|_{x=0}$

$$= f'(1+g(1)) \cdot (1+g'(1))$$
$$= \underbrace{f'(1+-4)}_{f'(-3)\text{ not possible}} \cdot \left(1+ -8/3\right)$$