9 Implicit Differentiation
1-9

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1. $-8x + 3y = -5$

$-8 + 3\frac{dy}{dx} = 0$

$3\frac{dy}{dx} = 8$

$\boxed{\frac{dy}{dx} = 8/3}$

2. $6y - 6x^2 = x - 6$

$6\frac{dy}{dx} - 12x = 1$

$6\frac{dy}{dx} = 12x + 1$

$\boxed{\frac{dy}{dx} = \frac{(12x+1)}{6}}$

$-8x \quad y$

$-8 \quad \frac{dy}{dx}$

3. $9y - 8xy - 7 = 0$

$9\frac{dy}{dx} - 8y - 8x\frac{dy}{dx} = 0$

$9\frac{dy}{dx} - 8x\frac{dy}{dx} = 8y$

$\frac{dy}{dx}(9 - 8x) = 8y$

$\boxed{\frac{dy}{dx} = \frac{8y}{9-8x}}$

4. $\cos(xy) + x^3 = y^3$

$x \quad y$

$1 \quad \frac{dy}{dx}$

$-\sin(xy)\left[y + x\frac{dy}{dx}\right] + 3x^2 = 3y^2\frac{dy}{dx}$

$-\sin(xy)\cdot y - \sin(xy)\cdot x\cdot\frac{dy}{dx} + 3x^2 = 3y^2\frac{dy}{dx}$

$-\sin(xy)\cdot y + 3x^2 = \sin(xy)\cdot x\cdot\frac{dy}{dx} + 3y^2\frac{dy}{dx}$

$3x^2 - \sin(xy)\cdot y = \frac{dy}{dx}(\sin(xy)\cdot x + 3y)$

$\boxed{\frac{dy}{dx} = \frac{3x^2 - y\cdot\sin(xy)}{x\cdot\sin(xy) + 3y}}$

5. $x^2 + y^2 - 2x + 4y = 8$ @ $(4,0)$

$2x + 2y\frac{dy}{dx} - 2 + 4\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 4\frac{dy}{dx} = 2 - 2x$

$\frac{dy}{dx}(2y + 4) = 2 - 2x$ ← everything div by 2

$\frac{dy}{dx} = \frac{1-x}{y+2}$

$\left.\frac{dy}{dx}\right|_{(4,0)} = \frac{1-4}{0+2} = \boxed{-3/2}$ ← tan slope

$2/3$ ← normal slope

$\boxed{\begin{array}{ll} y - 0 = -3/2\,(x-4) & \leftarrow \text{tan line} \\ y - 0 = 2/3\,(x-4) & \leftarrow \text{normal} \end{array}}$

$2x^2 \quad y$

$4x \quad \frac{dy}{dx}$

6. $2x^2y - \pi\cos y = 3\pi$ @ $(1,\pi)$

$4xy + 2x^2\frac{dy}{dx} + \pi\sin(y)\cdot\frac{dy}{dx} = 0$

$2x^2\frac{dy}{dx} + \pi\sin(y)\frac{dy}{dx} = -4xy$

$\frac{dy}{dx}(2x^2 + \pi\sin(y)) = -4xy$

$\frac{dy}{dx} = \frac{-4xy}{2x^2 + \pi\sin(y)}$

$\left.\frac{dy}{dx}\right|_{(1,\pi)} = \frac{-4(1)(\pi)}{2(1)^2 + \pi(\sin(\pi))} = \frac{-4\pi}{2+0}$

$= \boxed{-2\pi}$ ← tan slope

$1/2\pi$ ← normal slope

$\boxed{\begin{array}{ll} y - \pi = (-2\pi)(x-1) & \leftarrow \text{tan line} \\ y - \pi = (\frac{1}{2\pi})(x-1) & \leftarrow \text{normal line} \end{array}}$

$x \quad y$
$1 \quad \frac{dy}{dx}$

7. $xy - x + y = 5$

$y + x\frac{dy}{dx} - 1 + \frac{dy}{dx} = 0$

$x\frac{dy}{dx} + \frac{dy}{dx} = 1 - y$

$\frac{dy}{dx}(x+1) = 1 - y$

$\boxed{\frac{dy}{dx} = \frac{1-y}{x+1}}$

$1-y \quad x+1$
$-\frac{dy}{dx} \quad 1$

$\frac{-(1-y)}{x+1} = \frac{y-1}{x+1}$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{y-1}{\cancel{x+1}}\right)(\cancel{x+1}) - (1)(1-y)}{(x+1)^2}$$

$$= \frac{y-1-1+y}{(x+1)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2y-2}{(x+1)^2}}$$

8. $y^2 - x^2 = 4$

$2y\frac{dy}{dx} - 2x = 0$

$2y\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{x}{y}$

$x \quad y$
$1 \quad \frac{dy}{dx} \rightarrow \frac{x}{y}$

$$\frac{d^2y}{dx^2} = \left(\frac{y - x \cdot \frac{x}{y}}{y^2}\right)\frac{\cdot y}{\cdot y}$$

$$= \frac{y^2 - x^2}{y^3}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{4}{y^3}}$$

9. $y^2 = 2 + xy$

a) $2y \cdot \frac{dy}{dx} = y + x\frac{dy}{dx}$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx}(2y - x) = y$

$\boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$

(b) Slope of curve $= \frac{1}{2}$
tan slope $= 1/2$
derivative $= 1/2$

$(2y-x) \cdot 2 \cdot \frac{y}{2y-x} = \frac{1}{2} \cdot 2\,(2y-x)$

$2y = 2y - x$
$-2y \quad -2y$
$0 = x$

$y^2 = 2 + (0)y$

$y^2 = 2$

$y = \pm\sqrt{2}$

$\boxed{\text{Slope is } 1/2 \text{ at } (0, \sqrt{2}) \text{ and } (0, -\sqrt{2})}$

c) horizontal tan = 0 slope
0 slope = num is 0

$\frac{dy}{dx} = \frac{y}{2y-x} \rightarrow y = 0$

$0^2 = 2 + x(0)$

$0 \neq 2$

Since the only place on dy/dx the slope is horizontal is $y=0$ we substitute $y=0$ back into the original relation & find there is no place on the curve where $y=0$.

d) $2y\frac{dy}{dt} = y \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt}$

$2(3)(6) = (3)\frac{dx}{dt} + \left(\frac{7}{3}\right)(6)$

$36 = 3\frac{dx}{dt} + 14$

$22 = 3\frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = \frac{22}{3}}$

$x \quad y$
$\frac{dx}{dt} \quad \frac{dy}{dt}$

$(3)^2 = 2 + x \cdot 3$

$9 = 2 + x \cdot 3$

$7 = 3x$

$x = 7/3$