Calculus

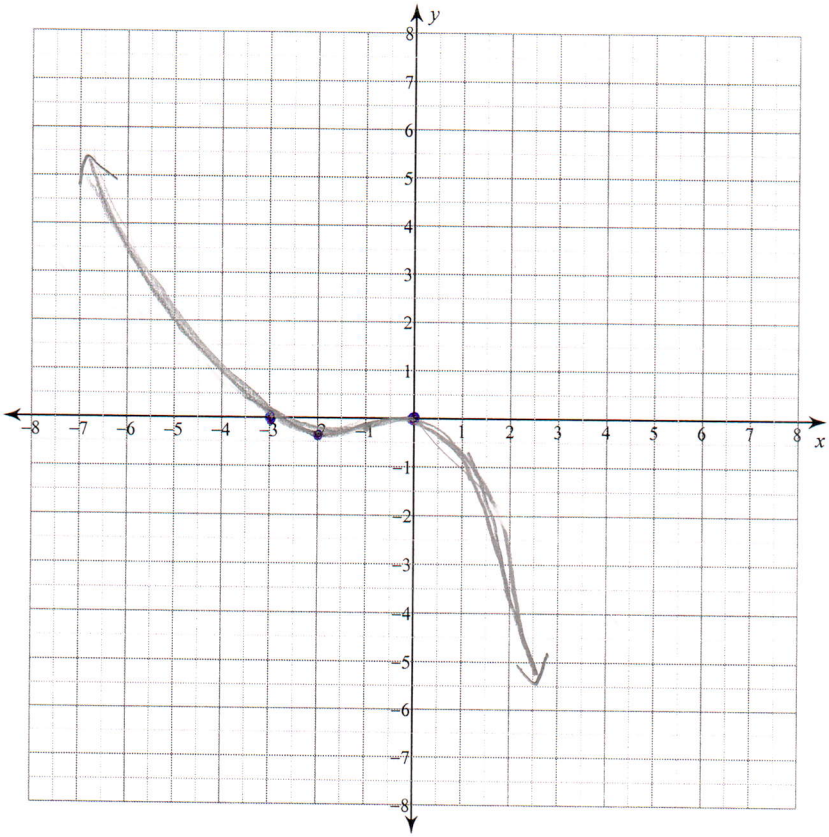
Name_____

17 Curve Sketching

Date_____ Period____

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\frac{x^3}{12} - \frac{x^2}{4}$



✓ x int $x = -3, 0$

✓ y int $y = 0$

✓ asymptotes N/A

✓ x-coord of CP. $f' = 0$ or undef $x = -2, 0$

Inc $(-2, 0)$

Dec $(-\infty, -2)$ $(0, \infty)$

CU $(-\infty, -1)$

CD $(-1, \infty)$

✓ max $x = 0$ $(0, 0)$

✓ min $x = -2$ $(-2, -1/3)$

CU

Inc

dec

CD

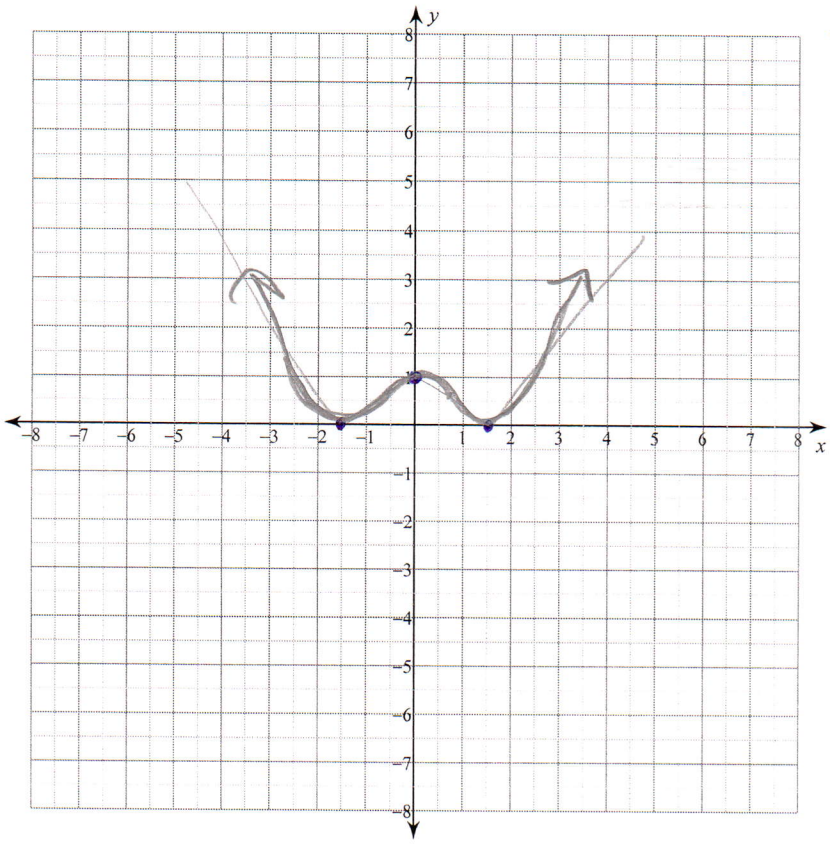
Inc

dec

Sketching

1. plot points (x, y-int, max/min)
2. Use inc/dec to draw straight lines
3. Use CU/CD to draw curves

2) $y = \frac{x^4}{4} - x^2 + 1$



✓ x-int $x = \pm\sqrt{2}$, $\approx \pm 1.4$

✓ y-int $(0,1)$

CP $x = \pm\sqrt{2}, 0$ $\pm 1.4, 0$

Inc: $(-\sqrt{2}, 0)$, $(\sqrt{2}, \infty)$

dec: $(-\infty, -\sqrt{2})$, $(0, \sqrt{2})$

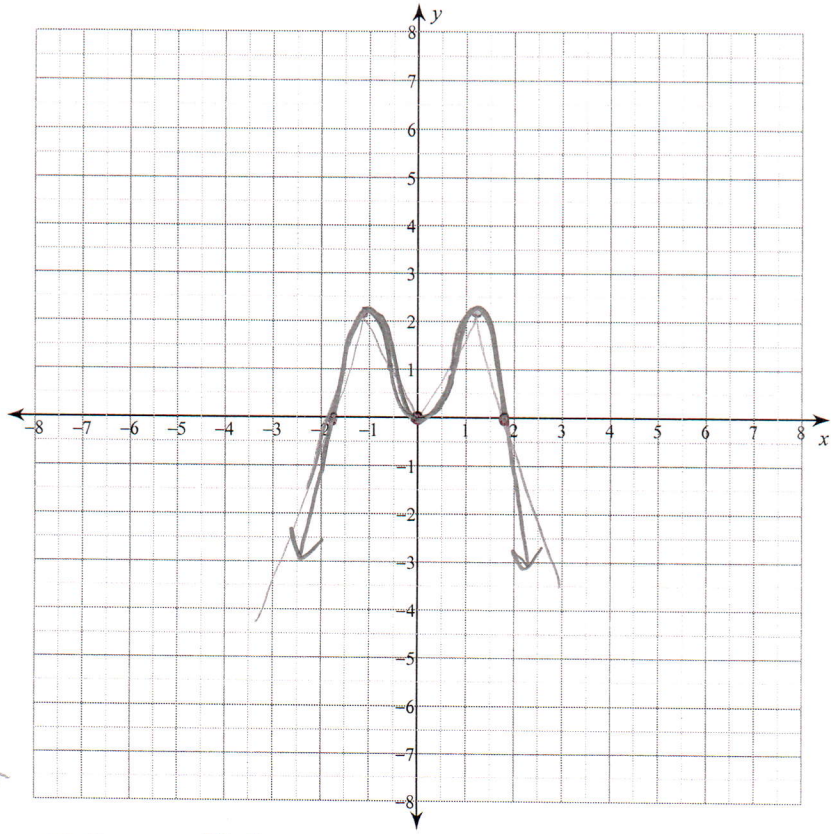
CU: $(-\infty, -.8)$, $(.8, \infty)$

CD: $(-.8, .8)$

✓ min $(-\sqrt{2}, 0)$ $(\sqrt{2}, 0)$

✓ max $(0,1)$

3) $y = -x^4 + 3x^2$



x-int $0 = -x^4 + 3x^2$

$0 = -x^2(x^2 - 3)$

$x = 0, \pm\sqrt{3}$

y int $y = -0^4 + 3 \cdot 0^2 = 0$

Asy: no denom $= 0$

CP $y' = -4x^3 + 6x$

$0 = -2x(2x^2 - 3)$

$x = 0, \pm\sqrt{\frac{3}{2}}$

max $y(-\sqrt{\frac{3}{2}}) = \frac{9}{4}$

$y(\sqrt{\frac{3}{2}}) = \frac{9}{4}$

y' + − + −

$-\sqrt{3/2}$ 0 $\sqrt{3/2}$

y inc dec inc dec

min $y(0) = 0$

Inc: $(-\infty, -\sqrt{3/2})$, $(0, \sqrt{3/2})$

dec: $(-\sqrt{3/2}, 0)$ $(\sqrt{3/2}, \infty)$

POI $y'' = -12x^2 + 6$

$0 = -12x^2 + 6$

$x = \pm\sqrt{1/2}$

y'' − + −

$-\sqrt{\frac{1}{2}}$ $\sqrt{\frac{1}{2}}$

y CD CU CD

CU: $(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}})$

CD: $(-\infty, -\sqrt{\frac{1}{2}})$, $(\sqrt{\frac{1}{2}}, \infty)$

max/min

$-(\sqrt{\frac{3}{2}})^4 + 3(\sqrt{\frac{3}{2}})^2$

$-\frac{9}{4} + \frac{9}{2} = \frac{-9}{4} + \frac{18}{4} = \frac{9}{4}$