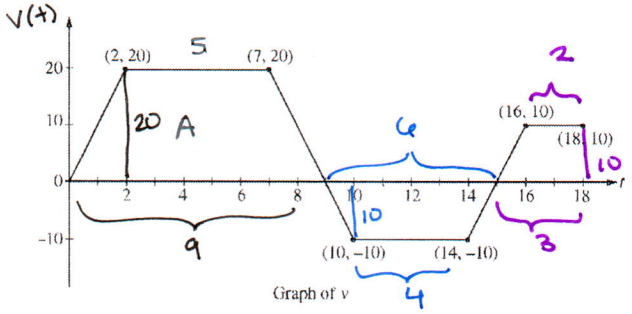
Key

On Friday Afternoon, I left the building quite late after working on your calculus reviews. As I walked to my car, I passed by the building and was struck in the head by a falling object. Sadly, it was my calculator costume, a cherished memory from my first year teaching Calculus. As I picked up the remains I discovered a note glued to one of the scarps of cardboard. It read:

Ms. Munch:

Lucky for you it was only cardboard. I've had just about enough of your calculus problems. Now I want you to solve some of ***mine***. We're going to play a little game. Follow orders exactly or you may end up in worse shape than your precious calculator. Here's the first question: I waited for you for 20 minutes to come by so I could drop this on you. Here is a graph of my velocity as I was pacing back and forth eagerly awaiting your arrival. Can you tell me the total distance I traveled in the first 18 minutes? Leave your reply under the brick beside your car.



Help me determine the identity of the perpetrator. Solve the previous problem in this space. To eliminate a suspect, sum the digits in the ones and tens place, then subtract the digit in the hundreds place and cross off the name that corresponds to that number on the list.

A: $\frac{9+5}{2} \cdot 20 = 140$

B: $\frac{6+4}{2} \cdot 10 = 50$

C: $\frac{2+3}{2} \cdot 10 = 25$

total dist $= 140 + 50 + 25 = 215$

$1 + 5 - 2 = 4$

Eliminated suspect: # 4

I was afraid **not** to solve the problem. I pulled a piece of paper from my bag, hurriedly scratched off a solution and located the brick. I hopped into my car and squealed out of the parking lot.

When I woke up the next morning, I decided the previous day's episode must have been a silly prank. So I returned to school to plan Monday's lesson. Of course, since it was Saturday after 1 pm, the building was locked so I had to work in the stinky old 700 building. It was an extremely hot day for April but the room was nice and cool inside. At least, it was cool in the room *at first*. But as I worked, I realized that the air conditioning must have been shut down because the room just got hotter and hotter by the minute. I tolerated it for about an hour and then, unable to stand it any longer, I packed up my things to leave. I tried to open the door of the room, but it was jammed. Then a note appeared under the door. Here's how it read:

Getting a mite warm in there isn't it? I've been keeping track of the temperature for you. Here is a little table of the readings that I've taken of the temp in the room starting at 1:00 pm when I shut off your AC.

t	Time (min)	0	10	20	30	40	50	60
$F(t)$	Temp (°F)	68	72.64	76.97	80	82.9	85.38	87.5

Use the trapezoidal rule to find the best approximation you can for the average temperature in the room from 1:00 until 2:00. You have 2 minutes to solve this problem. Slide your answer under the door or you'll be sweating for a looooooooooooooooooooong time.

Eliminate another suspect: It's the digit that appears in your answer twice. (Your answer should be rounded to three decimal places)

$$\text{Ave temp} = \frac{1}{60-0}\int_0^{60} F(t)\,dt$$

$$\text{Trap} \quad \int_0^{60} F(t)\,dt = \frac{68+72.64}{2}\cdot 10 + \frac{72.64+76.97}{2}\cdot 10 + \frac{76.97+80}{2}\cdot 10 + \frac{80+82.9}{2}\cdot 10$$

$$+ \frac{82.9+85.38}{2}\cdot 10 + \frac{85.38+87.5}{2}\cdot 10 = 4756.4$$

$$\text{Ave temp} = \frac{1}{60}\cdot 4756.4 = 79.2733$$

Eliminated suspect: # 7

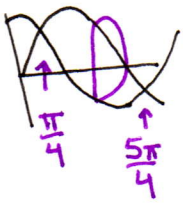
I solved the problem as quickly as I could and slid it under the door. But I guess I wasn't fast enough. Here's the note I got back:

> That was three minutes Munch. You blew it! But here's another chance...

At that point I started to panic. I thought to myself, "Maybe I can get through the window." But the window turned out to be jammed too, and I had nothing to break it with but my bare fist. Finally, the door clicked open. I looked out the window and saw a lone figure running off toward the track. I dropped everything, ran out the door, and made a beeline for my car. As I climbed into the driver's seat the door automatically locked! There on the steering wheel was ...I bet you can guess...another note! I turned the key and...nothing.

> I noticed you don't have a hood ornament on your car so I added one I made myself. Do you recognize it? It's a solid with a base formed by intersecting sine and cosine curves (between their first two points of intersection with positive x-values) and built up with semi-circular cross sections perpendicular to the x-axis. I want you to find two things:
>
> 1. The area of the base.
> 2. The volume of the solid.
>
> When you're done, add your answers and beep your horn that many times. If you are right, I will release the mechanism that has disabled your engine.

Eliminate two more suspects. You must find exact answers (radical, fraction, π, etc NOT DECIMALS) and then do the following.



$r = \frac{\sin x - \cos x}{2}$

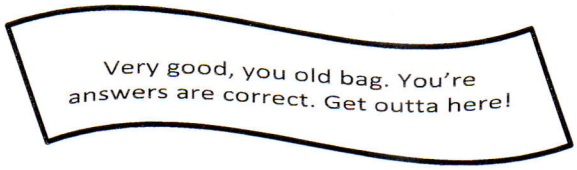
$A = \int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx = 2\sqrt{2}$

$V = \int_{\pi/4}^{5\pi/4} \frac{\pi r^2}{2} \, dx = \frac{\pi^2}{8}$

For question #1, take the value of the whole number outside the radical.
For question #2, take the denominator of your answer.

Eliminated suspect: # 2
Eliminated suspect: # 8

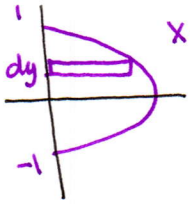
I worked furiously because I just wanted to get out of there. When I got my answers I beeped my horn and a big white flag unfurled from tree across the street where the chickens live. Here's what it said:



I ignored the incorrect use of "you're" because within seconds, the motor of my car turned on. I gunned the engine and sped off. Much to my dismay, the CD player came on automatically and I heard the words in a creepy Jigsaw-esque voice:

> Congratulations! You have a score equal to your IQ. You need a bigger challenge. Since you can't write and drive at the same time (or walk and chew gum either!) you have to do this one in your head. Look out the window of your car. You didn't have any identifying feature on your car's antenna so I carved a parabola out of the cover of your teacher's edition. Notice how nicely it spins around the antenna in the wind, you old windbag. Find the volume of this solid of revolution if the equation of the parabola is $x = 1 - y^2$ and the antenna is the y-axis. Write your answer on the windshield with lipstick. It better be there by the time you cross Tocayo or you might just end up in the Tijuana River going for a swim.

Eliminate another suspect. You must find an exact answer (radical, fraction, π, etc NOT DECIMALS). Divide your denominator by three to eliminate another suspect.



$x = 1 - y^2$

$$V = \pi \int_{-1}^{1} (1 - y^2)^2 \, dy = \frac{16\pi}{15}$$

$$\frac{15}{3} = 5$$

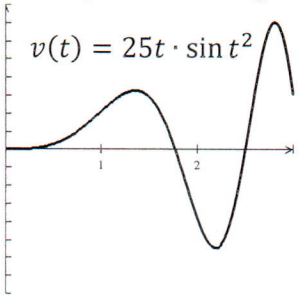
Eliminated suspect: # 5

This was easy for me because I had worked that very problem just the night before while I was doing pre-bedtime calculus, and I had the answer memorized. I fumbled through my purse for lipstick. I only had one color...Oompa Loompa Orange. I scribbled my answer on the windshield. I crossed Tocayo and then I heard a siren and saw flashing lights in my rearview mirror. The border patrol agent who pulled me over and said "Lady, are you drunk?!? Why are you writing numbers on your windshield in that beautiful color lipstick? " I told him I was being tormented by a lunatic calculus student who locked me in the nasty 700 building threw my giant calculator on me and "pimped" my ride. He said "step out of the vehicle and walk this line." After a humiliating couple of minutes walking backward and forward on the side of the road, the cop let me go with this warning, "Lay off the pills lady."

Finally, I arrived home and pulled into my driveway. Exhausted, I dragged myself up to the front door and found a note hanging on the knob. Here's what is said:

While you were walking the line for the cop, I was hiding in the bushes with my radar gun. I made a graph of your velocity in feet per minute during your little three minute jaunt. At time $t = 0$, you were backed up against the bumper of your car. Answer these three questions (correct to three decimal places) or your dogs, Hobbes and Chelsea, have fetched their last ball.

1. How far were you from your car two minutes after you started walking?
2. What was your acceleration 1 minute into the walk?
3. At what time was your instantaneous velocity the same as your average velocity over the first minute?



Eliminate another suspect: Take the first digit of the sum of the three answers

1. $\int_0^2 v(t)\,dt = \int_0^2 25t \cdot \sin(t^2)\,dt = 20.67054$

2. $a(t) = v'(t) = 25 \cdot \sin(t^2) + 50t^2 \cos(t^2)$

$a(1) = 48.05188$

$25t \quad \sin(t^2)$

$25 \quad \cos(t^2) \cdot 2t$

4. $v(t) = \frac{1}{1-0} \int_0^1 v(t)\,dt$

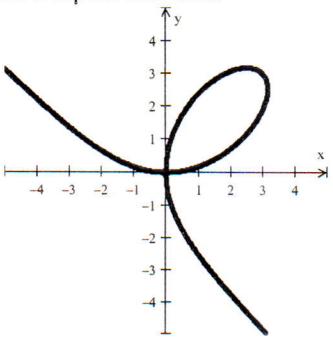
$v(t) = 5.74622$ from 0 to 1

$t = 0.61755$

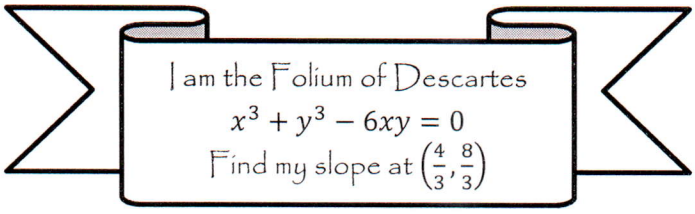
Eliminated suspect: # 6

$$\begin{array}{r} 20.67054 \\ 48.05188 \\ +\ 0.61755 \\ \hline 69.33997 \end{array}$$

I went inside and securely bolted the door. I was relieved to find nothing out of place and my precious dogs unharmed. I went into the kitchen for a cup of tea to soothe my nerves. I gulped it down and chased it with a delicious funnel cake. I sat down at the kitchen table and saw, for the first time, two packages. Kim buys a lot of stuff on amazon, but these didn't look like amazon packages, one was wrapped and tied with shiny pink ribbon shaped like this:



The other was shabbily constructed with aluminium foil. I was afraid to open either one. Then I saw the writing on the ribbon...



"This is ridiculous," I thought "How do they even know about the Folium?" I started to reach for the other package when I looked up to see that the culprit had drawn a mustache and devil horns on my beautiful oil painting, I couldn't help but think what EVIL student had done this to me.

$x^3 + y^3 - 6xy = 0$

$3x^2 + 3y^2\frac{dy}{dx} - 6y - 6x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$

$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$

$-6x \quad \times \frac{y}{\frac{dy}{dx}}$

-6

$\left.\frac{dy}{dx}\right|_{\left(\frac{4}{3}, \frac{8}{3}\right)} = \frac{2\left(\frac{8}{3}\right) - \left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2 - 2\left(\frac{4}{3}\right)} = \frac{\left(\frac{16}{3} - \frac{16}{9}\right)9}{\left(\frac{64}{9} - \frac{8}{3}\right)9}$

$= \frac{48 - 16}{64 - 24} = \frac{32}{40} = \frac{4}{5}$

Eliminate another suspect: Add the numerator and denominator of your answer.

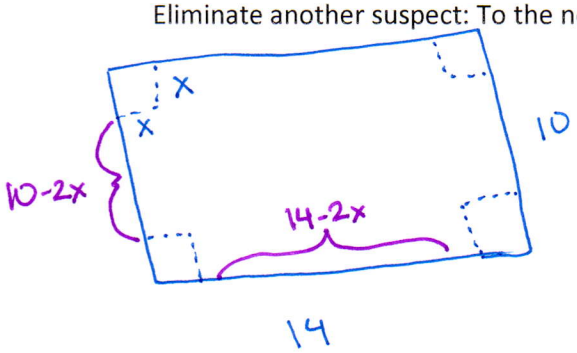
Eliminated suspect: # 9

4+5

So I worked the problem. I didn't know what would happen to me (or my dogs, perhaps my cat would be threatened next?) if I didn't do it. I taped my answer to the window and waited. Nothing happened, so I figured it was safe to open the other box. Under the aluminum foil I found this note:

> I filled this box with your favorite cookie – fig NEWTONs. Your pal Isaac sure was a busy little inventor! Of course, I made the box myself by cutting equal squares from the corners of a 10x14 inch rectangular piece of cardboard I took off you precious calculator before I threw it on you! I cut out the corners (measured to the nearest whole number of inches) so as to maximize the number of cookies I could fit into the box. Given that each fig newton is 2x1x.5 inches, how many fig newton's could I fit in the box? (assume each cookie must be fully intact)

Eliminate another suspect: To the nearest dozen, how many cookies are in the box?



$V = x(14-2x)(10-2x)$

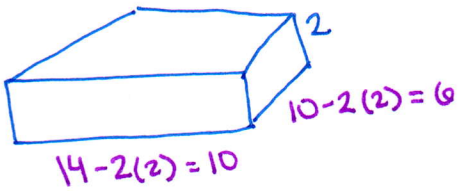
$V = 140x - 48x^2 + 4x^3$

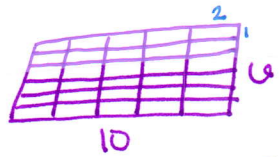
$V' = 140 - 96x + 12x^2$

$0 = 140 - 96x + 12x^2$

$x = 1.91833,\ 6.081067$ (too big for box)

$\hookrightarrow 2$ in for dimensions of box





30 cookies in a layer
x 4 layers
120 = 10 dozen

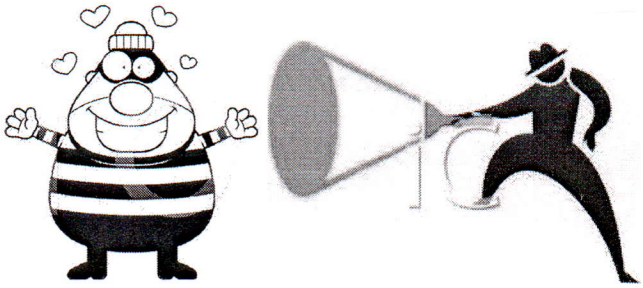
30 Cookies in a layer = .5 + .5 + .5 + .5 = 2 (4 layers)

Eliminated suspect: # 10

I was sick of this game. I knew the police would never believe me, and out of desperation I was gaining courage. I decided to take matters into my own hands. I turned out the lights and went upstairs to the balcony; I opened the door a wee bit and peered around the yard. No one was in sight but I was sure the punk was nearby. I exited quickly with my ninja skills and swiftly flipped over the balcony and down into the yard. With my excellent night vision I was able to make out the outline of a body up against the front wall of my yard. Being a ninja I knew all about estimating rates and distances.

I knew if a beam of flashlight were aimed at the wall it would form a cone whose altitude would change at the same rate that I approached the wall. If the volume of the cone of light was decreasing at a rate of 16 cubic feet per second, I figured the radius beam would increase at a rate of 7 ft/sec and the height decrease at 6 ft/sec. I asked myself "How far (to the nearest foot) from the scoundrel would I have to be for all of these facts to come together and illuminate him (or her??) in an 8 ft radius of light?"

$\frac{dV}{dt} = -$ $\frac{dr}{dt} =$ $\frac{dh}{dt} = -$ r $h = ?$



Use your answer to eliminate another suspect.

$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 \propto h$

$V' = \frac{2}{3}\pi r \cdot \frac{dr}{dt} h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$\frac{2}{3}\pi r \cdot \frac{dr}{dt} \quad \frac{dh}{dt}$

$-16 = \frac{2}{3}\pi(8)(7)h + \frac{1}{3}\pi(8)^2(-6)$

$-16 = \frac{112\pi}{3} \cdot h - 128\pi$

$(-16 + 128\pi)\left(\frac{3}{112\pi}\right) = h = 3.29215$

$\rightarrow 3$ ft

Eliminated suspect: # 3

I got within 5 ft and – to my dismay – I had miscalculated. My tormentor sprang up and ran off into the night. And that was the end of my ordeal.

I have related this tale to you, hoping that you might be able to help me discover the identity of my assailant. If you followed my tale and possess the mathematical skills to answer all the problems correctly, you will now know the identity of the culprit *and* be a true calculus nerd!

Who dun it? Suspect #1