FRF #2

1

AP Calculus
FRQ Planning/Solution Template

Name: Key
Date: ______ Per: ____

(a)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The function is cts because it meets the three Conditions for continuity:

① $\lim_{x\to 0} f(x) = 1$ ② $f(0) = 1$ ③ $\lim_{x\to 0} f(x) = f(0)$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f(x) = \begin{cases} 1-2\sin x & x \le 0 \\ e^{-4x} & x > 0 \end{cases}$$

$x=0$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

3 cond:

- limit exists (since piecewise check left and right)
- point exists (plug into $x=0$)
- those #'s agree

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

①
$$\lim_{x\to 0^-} (1-2\sin x) = 1 - 2\sin(0) = 1 - 0 = 1$$

$$\lim_{x\to 0^+} e^{-4x} = e^{-4(0)} = e^0 = 1$$

② $f(0) = 1 - 2\sin(0) = 1$

③ $\lim_{x\to 0} f(x) = f(0)$

$1 = 1$ ✓

(b)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The piecewise function $x \ne 0$ $f'(x) = \begin{cases} -2\cos x & x<0 \\ -4e^{-4x} & x>0 \end{cases}$

when $f'(x) = -3$, $x = \underline{-\frac{1}{4}\ln\frac{3}{4}}$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f(x) = \begin{cases} 1-2\sin x & x \le 0 \\ e^{-4x} & x > 0 \end{cases}$$

$x \ne 0$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

take derivative of pieces

$x \ne 0$ so remove $=0$ from boundary

Set BOTH pieces $= -3$ and solve (if possible)

$e^x = a$

$\ln a = x$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$$f'(x) = \begin{cases} 0 - 2\cos x & x<0 \\ e^{-4x}\cdot -4 & x>0 \end{cases}$$

$-2\cos x = -3$

$\cos x = 3/2$

Since $\cos x$ goes between 1 + -1, $\cos x \ne 3/2$ no x

$-4e^{-4x} = -3$

$e^{-4x} = 3/4$

$\ln 3/4 = -4x$

$x = -\frac{1}{4}\ln(3/4)$

FRF #2

1 cont

(C)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The average value of f on [-1,1] is $\frac{13}{8} - \cos(-1) - \frac{1}{8e^4}$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f(x) = \begin{cases} 1 - 2\sin x & x \le 0 \\ e^{-4x} & x > 0 \end{cases}$$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$$\frac{1}{1-{-1}} \int_{-1}^{1} f(x)\,dx = \frac{1}{2} \underbrace{\int_{-1}^{1} f(x)\,dx} = \frac{1}{2}\left[\frac{13}{4} - 2\cos(-1) - \frac{1}{4e^4}\right]$$

$$\int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4}x\,dx$$

$$= x + 2\cos x\Big]_{-1}^{0} + \frac{-1}{4}e^{-4x}\Big]_{0}^{1}$$

$$= \left[\underbrace{(0 + 2\cos(0))}_{2} - \underbrace{(-1 + 2\cos(-1))}_{+1 \; -2\cos(-1)}\right] + \left[\underbrace{\frac{-1}{4}e^{-4(1)}}_{-\frac{1}{4e^4}} - \underbrace{\frac{-1}{4}e^{-4(0)}}_{+\frac{1}{4}}\right]$$

$$= 3 - 2\cos(-1) - \frac{1}{4e^4} + \frac{1}{4}$$

$$= 3\tfrac{1}{4} - 2\cos(-1) - \frac{1}{4e^4}$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

ave value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

2 pieces → two integrals

boundary at $x=0$

1) Write a complete sentence answer with the actual solution blank.
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2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

FRF #2

2

(a)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

$f'(4) \approx -3$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$f(5) = -2$

$f(3) = 4$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$$f'(4) \approx \frac{f(5) - f(3)}{5-3} = \frac{-2-4}{5-3} = \frac{-6}{2} = -3$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

$f'(x)$ is a slope

estimate → 2 points

Close to $x=4$

(b)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

$\int_2^{13} (3 - 5f'(x))\,dx = 8$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$f(2) = 1$

$f(13) = 6$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$$\int (3 - 5\cdot f'(x))\,dx = \int 3 - \int 5\cdot f'(x)$$

$$\int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$$

$$= 3(13-2) - 5\left(f(13) - f(2)\right)$$

$$= 3(11) - 5(6-1)$$

$$= 33 - 25$$

$$= 8$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

Break apart Integral

Use areas for constant

and FTC

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

FRF #2

2 cont

(c)

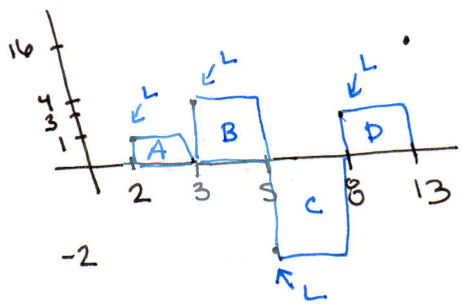
1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

$\int_2^{13} f(x)\,dx \approx 18$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

Use table for Riemann Sum

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.



A: $(1)f(2) = 1(1) = 1$
B: $(2)f(3) = 2(4) = 8$
C: $(3)f(5) = 3(-2) = -6$
D: $(5)f(8) = 5(3) = 15$

$\Big\}$ sum = 18

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

base = change in X
height = from Left side

(d)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

Tan line @ $x=5$ $\quad y_t = 3(x-5) - 2$

$f(7) \approx y_t(7) \le 4$

Sec line on $5 \le x \le 8$ $\quad y_s = 5/3(x-5) - 2$

$f(7) \approx y_s(7) \geq 4/3$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$f'(5) = 3$ Slope at $x=5$ is 3

$(5,-2)$ table

$f''(x) < 0$ CD $\rightarrow$ tan line is above graph

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

tan line $(5,-2)$ from table

$m = 3$ from given

$y_t = 3(x-5) - 2$

$y_t(7) = 3(7-5) - 2 = 6 - 2 = 4 \le 4$

Since tan line is above graph

sec line $[5,8]$ $\quad m = \dfrac{f(8) - f(5)}{8-5} = \dfrac{3 - -2}{8-5} = \dfrac{5}{3}$

at $(5,-2)$

$y_s = \frac{5}{3}(x-5) - 2$ or $\frac{5}{3}(x-8) + 3$

$y_s(7) = \frac{5}{3}(7-5) - 2 = \frac{10}{3} - 2 = \frac{10}{3} - \frac{6}{3} = \frac{4}{3} \geq 4/3$

Since sec line is below graph

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

CD $\quad y_t$ y_s • tan line above graph

• sec line below graph

$y_s(x) \le f(x) \le y_t(x)$

for $5 \le x \le 8$