

FRF #4

AP Calculus
FRQ Planning/Solution Template

Name: Key
Date: _____ Per: _____

(a)

- 1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

f has a critical point at $x = 4$, this is a max because $f'(x) > 0$ when $0 < x < 4$
max/min/neither $f'(x) < 0$ when $x > 4$

- 2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f'(x) = (4-x)x^{-3} \quad x > 0$$

↑
Not ≥

- 4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

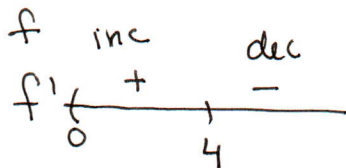
$$f'(x) = \frac{4-x}{x^3}$$

$$f'(x) = 0 \text{ when } 4-x=0$$

$$x=4$$

$$f'(x) \text{ DNE when } x^3=0$$

$$x=0 \leftarrow \text{Since } x > 0 \text{ this is NOT a CP.}$$



- 3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

CP $f' = 0$ or DNE
undefined

max inc to dec
min dec to inc
neither no change

(b)

- 1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The interval(s) on which f is concave down is/are $0 < x < 6$ because $f''(x) < 0$

- 2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f'(x) = \frac{4-x}{x^3} \quad x > 0$$

- 4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

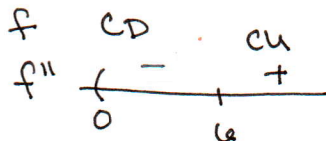
$$\begin{matrix} 4-x & \otimes & x^3 \\ -1 & & 3x^2 \end{matrix}$$

$$f''(x) = \frac{-x^3 - 3x^2(4-x)}{(x^3)^2}$$

$$= \frac{-x^3 - 12x^2 + 3x^3}{x^6} = \frac{2x^3 - 12x^2}{x^6}$$

$$0 = \frac{2x^2(x-6)}{x^6} = \frac{2(x-6)}{x^4}$$

$$x = 6, 0$$



- 3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

Concave down $f'' < 0$
quotient rule $\frac{f}{g} \rightarrow \frac{f'g - g'f}{g^2}$

FRF #4

1 cont

(C)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

$$f(x) = \frac{-2}{x^2} + \frac{1}{x} + 3$$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$f'(x) = \frac{4}{x^3} - \frac{1}{x^2}$$

$$f(1) = 2$$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$$f'(x) = 4x^{-3} - x^{-2}$$

$$\int f'(x) = \int 4x^{-3} - x^{-2} dx$$

$$f(x) = \frac{4x^{-2}}{-2} - \frac{x^{-1}}{-1} + C$$

$$f(1) = -2(1)^{-2} + (1)^{-1} + C = 2$$

$$= -2 + 1 + C = 2$$

$$C = 3$$

$$f(x) = \frac{-2}{x^2} + \frac{1}{x} + 3$$

$$\begin{aligned} f(x) &= f(1) + \int_1^x 4t^{-3} - t^{-2} dt \\ &= 2 + \left[-2t^{-2} + t^{-1} \right]_1^x \\ &= 2 + \frac{-2}{x^2} + \frac{1}{x} - (-2+1) \\ &= 3 + \frac{-2}{x^2} + \frac{1}{x} \end{aligned}$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

$$\int f'(x) = f(x) + C$$

Separate
Integrate

Constant of integration

Initial condition

Solve for y

1) Write a complete sentence answer with the actual solution blank.
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2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

FRQ #4

2

(a)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The approximate acceleration of the particle at 36s is

$\frac{11}{8} \text{ m/s}^2$

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$v(40) = 7$
 $v(32) = -4$

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

$a(36) = v'(36) \approx \frac{v(40) - v(32)}{40 - 32} = \frac{7 - (-4)}{8} = \frac{11}{8}$

$\frac{\text{m/s}}{\text{s}} \rightarrow \text{m/s}^2$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

$\text{accel} = \frac{\Delta v}{\Delta t}$

approximate / estimate

\rightarrow 2 points close to $t = 36$

(b)

1) Write a complete sentence answer with the actual solution blank.
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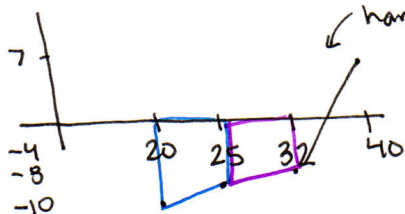
$\int_{20}^{40} v(t) dt = -75 \text{ m}$, this is particle's change in position from 20s to 40s. *units!*

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

use table data

Start = 20
Stop = 40

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.



3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

$\int v(t) = \text{position}$

trapezoid

$\text{y-value} \left(\frac{b_1 + b_2}{2} \right) \cdot h$
 \uparrow
 Δx

$\frac{(-10 + -8)}{2} \cdot 5 + \frac{(-8 + -4)}{2} \cdot 7 + \frac{(-4 + 7)}{2} \cdot 8$

$= -45 + -42 + 12 = -75$

(c)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

Yes, the particle must change direction in the interval y/n must/not

because the sign of the velocity changes

y → The particle changes direction in the interval $(8, 20) + (32, 40)$

n → Since

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

table data

$$v(8) > 0 \quad v(20) < 0$$

$$v(32) < 0 \quad v(40) > 0$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

Change direction

means velocity changes

signs

t	0	8	20	25	32	40	
v	+	+	-	-	-	+	
		R	R	L	L	L	R

↗ ↘ ↗ ↘ ↗ ↘
direction change

(d)

1) Write a complete sentence answer with the actual solution blank.
5) Fill in the blank spot to complete the solution.

The position of the particle must be greater than 30m because the particle starts at $x=7m$ and if the slowest the particle traveled for 8 hrs was 3 m/s, it gains 24m > 30m

4) Solve the problem. Make use of your strategy and given information. If you find you need more info, go back and revise.

2) Write down any given information that will/may be useful. If you end up using something you didn't write down, come back and include it.

$$a > 0 \quad (0, 8)$$

$$x(0) = 7 \quad \leftarrow \text{problem}$$

3) Write down your strategy. Include any definitions, alternate meanings, steps, or things to exclude.

$a > 0$ means

$v'(t) > 0$
 So v is increasing
 from $t=0$ to $t=8$
 $\leftarrow v$ starts at 3 m/s

pos = start pos + accumulated position

$$x(8) = x(0) + \int_0^8 v(t) dt$$

$$= 7 + \int_0^8 v(t) dt \geq 7 + 24 = 31 > 30$$

Smallest v is 3 so for 8 hours

$$\int_0^8 v(t) dt \geq 8 \cdot 3$$