

## L'Hopital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

← for  $\frac{0}{0}$  or  $0 \cdot \infty$  or  $0 \cdot \text{undef}$

Evaluate each limit using L'Hôpital's Rule.

1)  $\lim_{x \rightarrow \infty} \frac{x}{e^{2x}} = \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{1}{2e^{2x}} = \frac{1}{\infty} = \boxed{0}$$

2)  $\lim_{x \rightarrow 0} \frac{\tan(3x)}{5x} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{3 \sec^2(3x)}{5} = \frac{3(1)^2}{5} = \boxed{\frac{3}{5}}$$

3)  $\lim_{x \rightarrow 0^+} x^2 \ln x = 0 \cdot (-\infty)$

\* let  $t = \frac{1}{x} \rightarrow x = \frac{1}{t}$  } trick  
 $x \rightarrow 0^+$  means  $t \rightarrow \infty$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{t}\right)^2 \ln\left(\frac{1}{t}\right)$$

$$\lim_{t \rightarrow \infty} \frac{\ln(t^{-1})}{t^2}$$

$$\lim_{t \rightarrow \infty} \frac{-\ln(t)}{t^2} = \lim_{t \rightarrow \infty} \frac{-\frac{1}{t}}{2t} = \lim_{t \rightarrow \infty} \frac{-1}{2t^2} = \boxed{0}$$

4)  $\lim_{x \rightarrow 0} \frac{3(e^x - e^{-x})}{x} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{3(e^x + e^x)}{1} = \frac{3(1+1)}{1} = \boxed{6}$$

5)  $\lim_{x \rightarrow 0} 3x \cot x = 0 \cdot \text{und}$

$$\lim_{x \rightarrow 0} \frac{3x \cos x}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{3 \cos x - 3x \sin x}{\cos x}$$

$$= \frac{3(1) - 0}{1} = \boxed{3}$$

6)  $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2}-1} = \frac{0}{0}$

$$= \lim_{x \rightarrow 3} \frac{1}{\frac{1}{2\sqrt{x-2}}} = \frac{1}{\frac{1}{2\sqrt{1}}} = \boxed{2}$$

## L'Hopital's Rule

Evaluate each limit using L'Hôpital's Rule.

1) 
$$\lim_{x \rightarrow 0} \frac{5(e^x - e^{-x})}{\sin x}$$

2) 
$$\lim_{x \rightarrow 1} \frac{(x-1)}{\ln x}$$

3) 
$$\lim_{x \rightarrow 1} \frac{2 \ln x}{x-1}$$

4) 
$$\lim_{x \rightarrow -2} \frac{x^2 + 3x + 2}{x + 2}$$

5) 
$$\lim_{x \rightarrow 0^+} 4x \ln x$$

6) 
$$\lim_{x \rightarrow 1} \frac{4 \ln x^2}{x^2 - 1}$$