Limits: An Analytical Approach
1-25

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1. $\lim\limits_{x\to 5} 12 = 12$

2. $\lim\limits_{x\to 0} \pi = \pi$

3. $\lim\limits_{x\to 2} 4x = 4(2) = 8$

4. $\lim\limits_{x\to 5} 3x^2-4x-1 = 3(5)^2-4(5)-1 = 54$

5. $\lim\limits_{x\to 0^-} 5x^3-7x^2+2x-2 = 5(0)^3-7(0)^2+2(0)-2 = -2$

6. $\lim\limits_{y\to -1} 3y^4-6y^3-2y = 3(-1)^4-6(-1)^3-2(-1) = 11$

7. $\lim\limits_{x\to 4} \frac{2x-4}{x-1} = \frac{2(4)-4}{4-1} = 4/3$

8. $\lim\limits_{x\to -2} \frac{x^2+4x+4}{x^2} = \frac{(-2)^2+4(-2)+4}{(-2)^2} = 0/4 = 0$

9. $\lim\limits_{x\to 1} \frac{2x-2}{x-1} = \lim\limits_{x\to 1} \frac{2\cancel{(x-1)}}{\cancel{(x-1)}} = \lim\limits_{x\to 1} 2 = 2$

10. $\lim\limits_{x\to 4} \frac{x^2-16}{x-4} = \lim\limits_{x\to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}} = \lim\limits_{x\to 4} x+4 = 4+4 = 8$

11. $\lim\limits_{t\to -2} \frac{t^3+8}{t+2} = \lim\limits_{t\to -2} \frac{\cancel{(t+2)}(t^2-2t-4)}{\cancel{(t+2)}} = \lim\limits_{t\to -2} t^2-2t-4 = (-2)^2-2(-2)-4 = 4$

12. $\lim\limits_{x\to 2} \frac{x^2-4x+4}{x^2+x-6} = \lim\limits_{x\to 2} \frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x+3)} = \lim\limits_{x\to 2} \frac{(x-2)}{(x+3)} = \frac{2-2}{2+3} = 0/5 = 0$

13. $\lim\limits_{x\to -1} \frac{(x^2+6x+5)}{x^2-3x-4} = \lim\limits_{x\to -1} \frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x-4)} = \lim\limits_{x\to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4} = 4/-5 = -4/5$

14. $\lim\limits_{x\to 1} \frac{x^3+x^2-5x+3}{x^2-3x+2} = \lim\limits_{x\to 1} \frac{\cancel{(x-1)}(x^2+2x-3)}{\cancel{(x-1)}(x-2)}$

$= \lim\limits_{x\to 1} \frac{x^2+2x-3}{x-2} = \frac{1^2+2(1)-3}{1-2} = \frac{0}{-1} = 0$

$$\begin{array}{r} x^2+2x-3 \\ x-1\,\overline{)\,x^3+x^2-5x+3} \\ -(x^3-x^2) \\ \hline 2x^2-5x \\ -(2x^2-2x) \\ \hline -3x+3 \\ -(-3x+3) \\ \hline 0 \end{array}$$

15. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

$$\left(\frac{\sqrt{2+x}-\sqrt{2}}{x}\right)\left(\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right) = \frac{(2+x)-(2)}{x(\sqrt{2+x}+\sqrt{2})}$$

$$= \frac{x}{\cancel{x}(\sqrt{2+x}+\sqrt{2})}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

16. $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$

$$\left(\frac{\sqrt{3+x}-\sqrt{3}}{x}\right)\left(\frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}\right) = \frac{(3+x)-(3)}{x(\sqrt{3+x}+\sqrt{3})} = \frac{x}{\cancel{x}(\sqrt{3+x}+\sqrt{3})}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{\sqrt{3+0}+\sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}}$$

17. $\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$$\frac{\left(\frac{1}{x+4}-\frac{1}{4}\right)}{x}\left(\frac{4(x+4)}{4(x+4)}\right) = \frac{4-(x+4)}{x(4(x+4))} = \frac{4-x-4}{x(4(x+4))}$$

$$= \frac{-x}{x(4)(x+4)} = \frac{-1}{4(x+4)}$$

$$= \lim_{x\to 0} \frac{-1}{4(x+4)} = \frac{-1}{4(0+4)}$$

$$= -\frac{1}{16}$$

19. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x}$

$$\frac{(x+\Delta x)^2-x^2}{\Delta x} = \frac{\cancel{x^2}+2x(\Delta x)+(\Delta x)^2-\cancel{x^2}}{\Delta x}$$

$$= \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}} = 2x+\Delta x$$

$$= \lim_{\Delta x\to 0} 2x+\Delta x = 2x+0$$

$$= 2x$$

18. $\lim_{\Delta x\to 0} \frac{2(x+\Delta x)-2x}{\Delta x} = \lim_{\Delta x\to 0} \frac{\cancel{2x}+2\Delta x-\cancel{2x}}{\Delta x} = \lim_{\Delta x\to 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x\to 0} 2 = 2$

20. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

$$= \left[(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)\right]/\Delta x$$

$$= \left[x^2+2x(\Delta x)+(\Delta x)^2-2x-2\Delta x+1-x^2+2x-1\right]/\Delta x$$

$$= \frac{\cancel{\Delta x}(2x+\Delta x-2)}{\cancel{\Delta x}}$$

$$= 2x+\Delta x-2$$

$$\lim_{\Delta x\to 0} 2x+\Delta x-2 = 2x+0-2 = 2x-2$$

21. $\lim_{x\to 0} \frac{\sin x}{5x} = \frac{1}{5} \cdot \lim_{x\to 0} \frac{\sin x}{x} = \frac{1}{5} \cdot 1 = 1/5$

22. $\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \sin x \cdot \left(\frac{\sin x}{x}\right) = \underbrace{\lim_{x\to 0} \sin x}_{0} \cdot \underbrace{\lim_{x\to 0} \left(\frac{\sin x}{x}\right)}_{1} = 0$

23. $\lim_{x\to 0} \frac{3(1-\cos x)}{x} = 3 \cdot \lim_{x\to 0} \frac{(1-\cos x)}{x} = 3 \cdot 0 = 0$

24. $\lim_{t\to 0} \frac{\sin 3t}{t} = \lim_{t\to 0} \frac{3 \cdot \sin 3t}{3t} = 3 \cdot \lim_{t\to 0} \frac{\sin 3t}{3t} = 3 \cdot 1 = 3$

25. Table: Evaluate a function at values VERY close to your desired x value, estimate using the left + right side of the desired x value to obtain the approached Y-value.

Graph: Visually inspect where the graph (y-value) SHOULD BE based on y-values approaching a specified x-value.

Algebraic: 1st Plug in! If you obtain an undefined value $\left(\frac{0}{0}\right)$ utilize algebraic manipulations (factoring, conjugate, common denominator, reducing etc) to eliminate the $\frac{0}{0}$ cause. Then plug in your desired x-value.

All of these methods relate approaching an x-value to obtain an "approached" y-value, should one exist.