4 Asymptotes: Limits involving Infinity

Vertical Asymptotes:

- very LARGE values of Y
- Denominator =0 and WILL NOT reduce

$$\lim_{x \to a} f(x) = \pm\infty$$

Horizontal Asymptotes:

- very large values of X
- ratio of the largest power of x

$$\lim_{x \to \pm\infty} f(x) = a$$

Vertical Asymptotes

$$\lim_{x \to a} f(x) = \pm\infty$$

We get a vertical asymptote when we evaluate the limit and obtain an undefined value.

Undefined = + or - ∞

We simply need to determine **WHICH** infinity the function is going toward.

To do this we use the left and right hand limits and determine the SIGN of the limit evaluated **VERY SLIGHTLY** to the left or right.

e.g.

If we plug in a number SLIGHTLY smaller than 2, is (x-2) positive or negative?

What about a number slightly larger than 2?

Horizontal Asymptotes

$$\lim_{x \to \pm\infty} f(x) = a$$

We get a horizontal asymptote when we look at X values that are very large. There are three possibilities for a horizontal asymptote.

$$\lim_{x \to \pm\infty} f(x) = 0$$

function is "bottom heavy" meaning the LARGER/faster function is on the bottom

$$\lim_{x \to \pm\infty} f(x) = L$$

usually polynomial rational functions, powers are the same in numerator and denominator

$$\lim_{x \to \pm\infty} f(x) = DNE$$

function is "top heavy" meaning the LARGER/faster function is on the top.

Which function is "faster" as x gets very large?

$$e^x \quad vs \quad 2^x$$

2.71 x 2

Ctrl g

$$x^2 \quad vs \quad x^3$$

$$e^x \quad vs \quad \log x$$

$$\frac{\log x}{e^x}$$

Limits Involving infinity

Ex 1) $\lim_{x \to 2} \frac{1}{x-2} =$ DNE VA or HA

$\frac{1}{0}$

$$\lim_{x \to 2^-} \frac{1}{x-2} = \frac{1}{-\#} = -\infty$$

$$\lim_{x \to 2^+} \frac{1}{x-2} = \frac{1}{+\#} = \infty$$

Ex 2) $\lim_{x \to 2} \frac{2x+5}{x-2}$ $= f(x)$

VA or HA

1) Solve by hand

$$\lim_{x \to 2^-} f(x) = \frac{+}{-} = -\infty$$

$$\lim_{x \to 2^+} f(x) = \frac{+}{+} = +\infty$$

2) Use <u>calc</u> to solve limit from <u>left</u>

3) Graph the fn

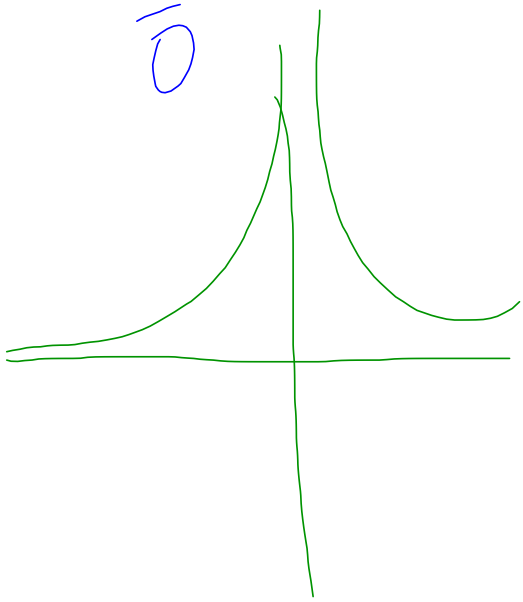
Ex 3) $$\lim_{x \to 0} \frac{4}{x^2}$$

VA or HA

$\frac{\#}{0}$

$$\lim_{x \to 0^-} \frac{4}{x^2} = +\infty$$

$$\lim_{x \to 0^+} \frac{4}{x^2} = \infty$$



Ex 4) VA or HA

$$\lim_{x\to -3} \frac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(x+3)}$$

$$\lim_{x\to -3^-} \frac{x-1}{x+3} = \frac{-}{-} = +\infty$$

$$-3^+ \qquad = \frac{-}{+} = -\infty$$

Ex 5)

VA or HA

$$\lim_{x \to 5} \frac{x}{x-5} = \frac{5}{0} = \text{DNE}$$

Undef $\Rightarrow \pm\infty$

$$\lim_{x \to 5^-} f(x) = \frac{+}{-} = -\infty$$

$$\lim_{x \to 5^+} f(x) = \frac{+}{+} = \infty$$

Ex 6) $\lim_{x \to 5} \frac{x}{(x-5)^2} = \frac{5}{0} = \infty$ VA or HA

$\lim_{x \to 5^-} f(x) = \frac{+}{+} = \infty$

$\lim_{x \to 5^+} f(x) = \frac{+}{+} = \infty$

Ex 7) $\lim_{x \to -3} \frac{x+1}{x+3}$ $= \frac{-2}{0}$ VA or HA

$= DNE$

$\lim_{x \to -3^-} \frac{x+1}{x+3} = \frac{-}{-} = \infty$

-3^-

$\lim_{x \to -3^+} \frac{}{-3^+ + 3} = \frac{-}{+} = -\infty$

Ex 8)

VA or HA

$$\lim_{x\to\infty}\frac{4x^2+50}{x^3-85}$$

$$\lim_{x\to\infty}\frac{0x^3+4x^2+50}{1x^3-85}=0$$

$$\frac{1}{1}\quad\frac{1}{10}\quad\frac{1}{100}\quad\frac{1}{1000}$$

Ex 9)

VA or HA

$$\lim_{x\to\infty}\frac{4x^3-5x^2+3x-1}{5x^3-7x-25} = \frac{4}{5}$$

Limits Involving infinity

Ex 10)

$$\lim_{x\to\infty}\frac{3x^3-23}{4x-1}$$

VA or HA

$$\lim_{x\to\infty}\frac{3x^3-23}{0x^3+4x-1} = \text{Undef}$$

Ex 11) $\lim_{x \to \infty} \frac{\ln x}{e^x} = 0$ VA or HA

$e^x > \ln x$

bottom heavy

Ex 12) VA or HA

$$\lim_{x\to\infty}\frac{2^x}{3^x} = 0$$

$$\frac{2x}{3x}$$?'s

ok HA or VA? why HA?

so which is "faster"? top or **bottom**

2*2

3*3

So we can see the bottom gets bigger faster yes?

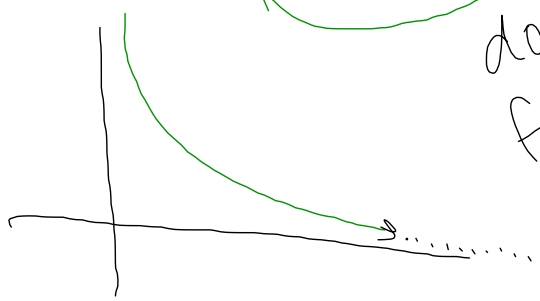
$$\frac{2^x}{3^x} = \left(\frac{2}{3}\right)^x$$

r

>1

this is growth or <1 decay?

exponential fn



closer to zero

do we remember from MA?

Limits Involving infinity

2^x & 3^x

are not growing at the <u>same rate</u>

?'s

$2x$ & $3x$

are goal?

Ex 13) $\lim_{x \to 2} \frac{2^x}{3^x} = \frac{4}{9}$

VA or HA ?

is it an asymptote?

What did I tell you to do 1st w/ limits?

$\frac{2^2}{3^2} = \frac{4}{9}$

is this an asymptote?

if $x \to \pm\infty$ → HA

if $\lim \frac{\text{num}}{\text{den}} = \pm\infty$ VA

Don't forget that you need to be able to go from one type of limit to another

- plug in
- factor/reduce etc
- asymptote (HA or VA)

You need to be looking for all of them

Ex 14)

$$\lim_{x\to\infty} \frac{\sqrt{x^2 \cancel{-3x}}}{2x \cancel{+1}}$$

VA or HA

$+\infty$

so now you do this problem

what doesn't matter

$$\approx \lim_{x\to\infty} \frac{\sqrt{x^2}}{2x} \approx \lim_{x\to\infty} \frac{|x|}{2x}$$

x

$+$

$$= \frac{1}{2}$$

do you have?

Limits Involving infinity

Ex 15) VA or HA

$$\lim_{x\to-\infty}\frac{\sqrt{x^2-3x}}{2x+1}$$

why?

$$\lim_{x\to-\infty}\frac{|x|}{2x}$$

$$=-\frac{1}{2}$$

red graph

$$\frac{|x|}{2x}$$

any ?'s

usually square roots means 2 HA's

not always

Limits Involving infinity

Ex 16) $\lim_{x \to 0} \frac{x}{|x|}$ = why not? ?

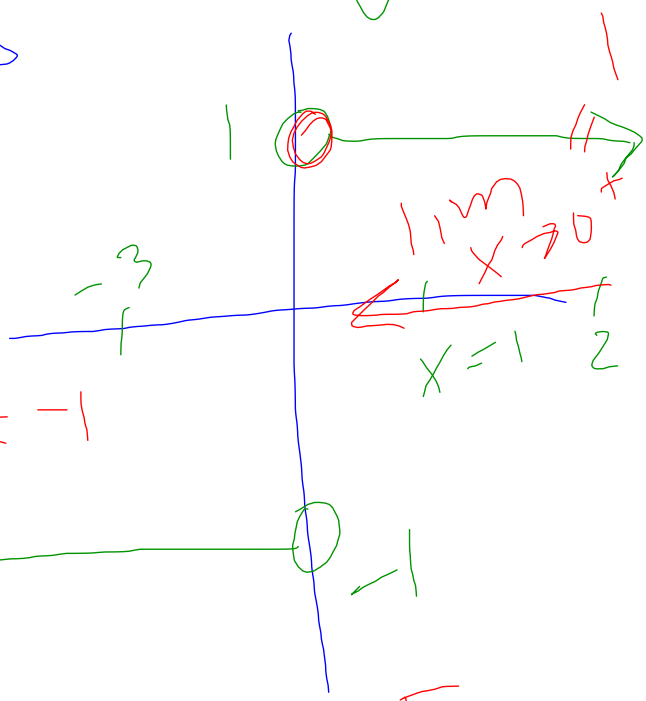
VA or HA

why $x \to 0$

graph

1st thing to do w/ lims

$\frac{0}{0}$



$\lim_{x \to 0} \frac{x}{|x|}$ = DNE

why?

LH ≠ RH ?

Limits Involving infinity

Ex 17)

VA or HA

$$\lim_{x\to-\infty}\frac{2x+\cancel{7}}{|x|-\cancel{24}} =$$

$-\infty$

What doesn't matter as x gets big?

$$\approx \lim_{x\to-\infty}\frac{2x}{|x|} = -2$$

$-\#$

$+\#$

does one grow faster than the other or are they the same power?

Assignment #4 Due:

~~Friday 8/23~~

Monday 8/26

Limits Involving infinity