Limits: Tabular (table) and Graphical Approach

Limit: where a function is supposed to get to.

A Limit is a **Y** value

as x approaches (get VERY close) to a certain number, what is the y value SUPPOSED to be

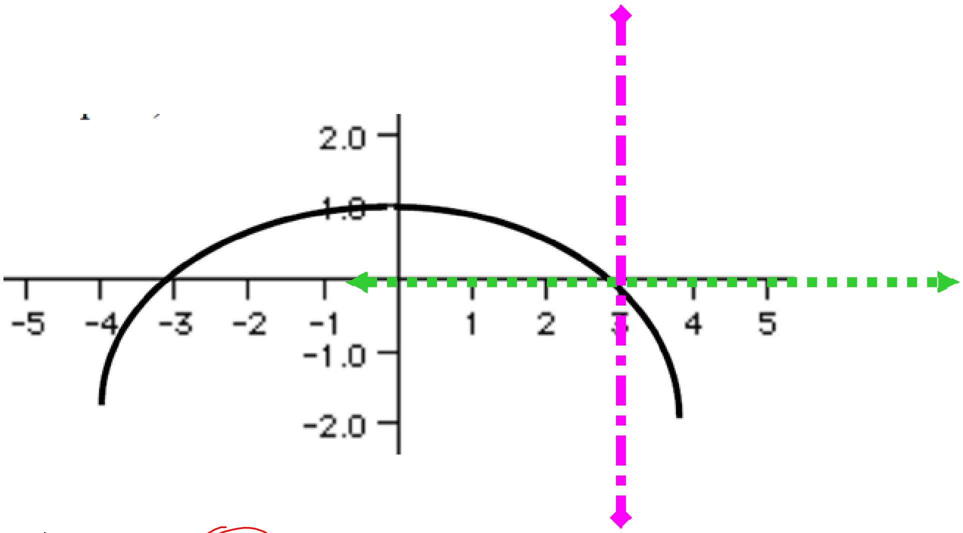
what is happening to the y-value when x gets close to a certain value

$$\lim_{x \to a} f(x) =$$

"the limit of f(x) as x approaches a is"

"the limit as x approaches a of f(x) is"

Graphical Approach



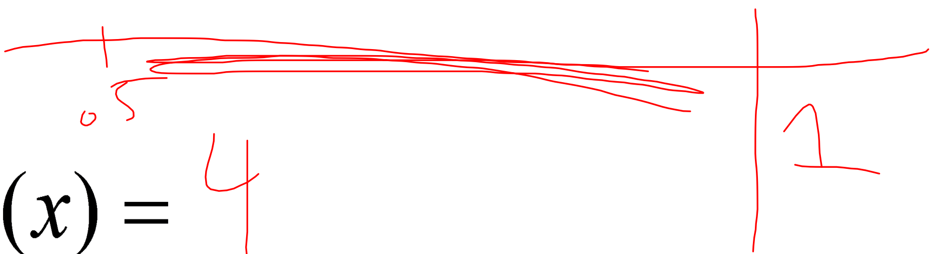
$$\lim_{x \to -3} f(x) = 0$$

$$\lim_{x \to 0} f(x) = 1$$

$$\lim_{x \to 3} f(x) = 0$$

Tabular Approach

x	.5	.8	.9	.99	.999	1
f(x)	3.5	3.8	3.9	3.99	3.999	?



$$\lim_{x \to 1^-} f(x) = 4$$

f(1)

Different types of limits

Left hand limit $\lim_{x \to a^-} f(x) =$

approaching from the LEFT

table looks at numbers **less than** a

Right hand limit $\lim_{x \to a^+} f(x) =$

approaching from the RIGHT

table looks at numbers **greater than** a

In order for a LIMIT to exists, both sides must AGREE!!

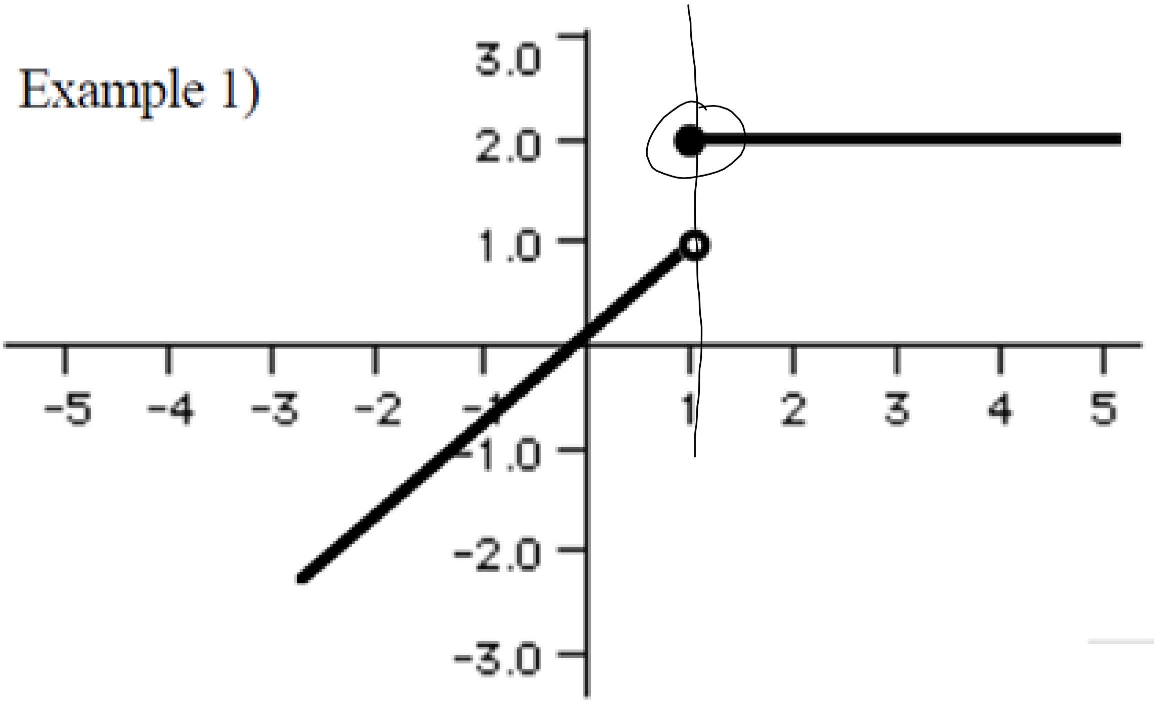
Three causes for DNE (does not exist):

1) LHL≠RHL
2) Oscillating behavior (like sin)
3) Unbounded (±∞)

keep in mind, ±∞ are not real numbers, infinity is a description, an idea, so TECHNICALLY a limit can be DNE and infinity...

BUT our goal is to give the best DESCRIPTION!!

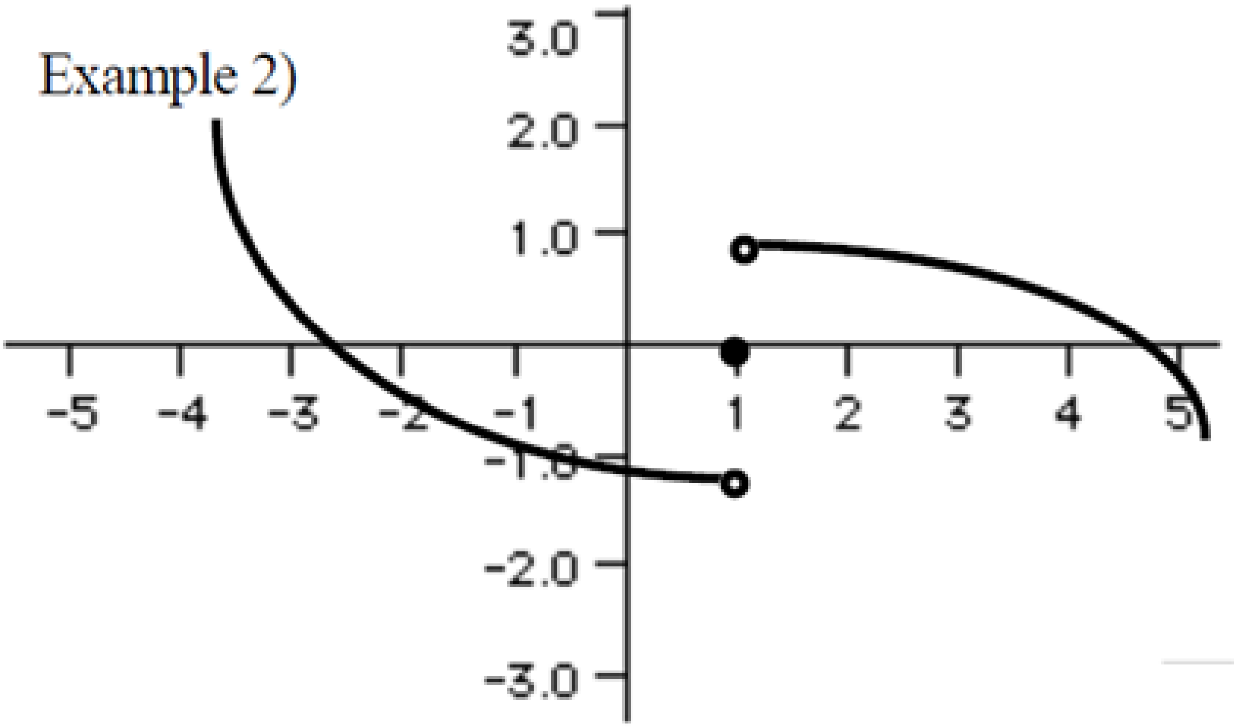
Limits: Tabular and Graphical approach



$$\lim_{x \to 1^-} f(x) = 1 \qquad \lim_{x \to 1^+} f(x) = 2$$

$$\lim_{x \to 1} f(x) = \text{DNE} \qquad f(1) = 2$$

Limits: Tabular and Graphical approach

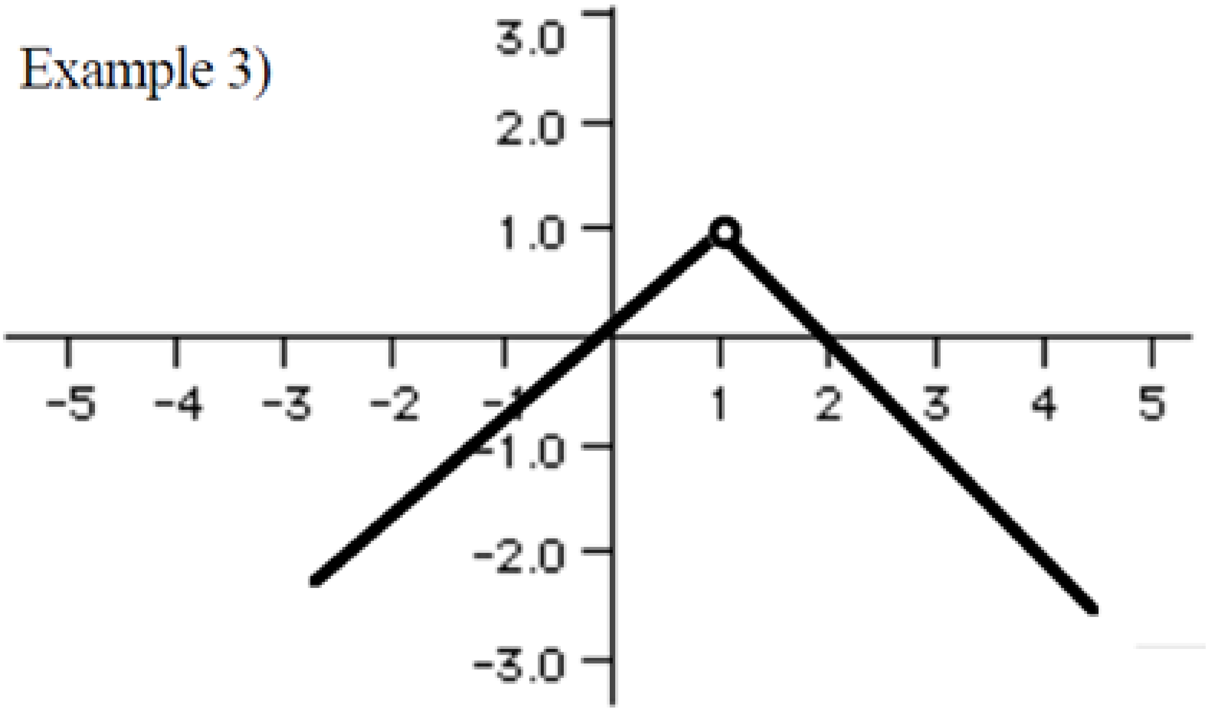


$$\lim_{x \to 1^-} f(x) = -1 \qquad \lim_{x \to 1^+} f(x) = 1$$

$$\lim_{x \to 1} f(x) = \text{DNE} \qquad f(1) = 0$$

Limits: Tabular and Graphical approach

Example 3)



$$\lim_{x\to 1^-} f(x) = 1$$

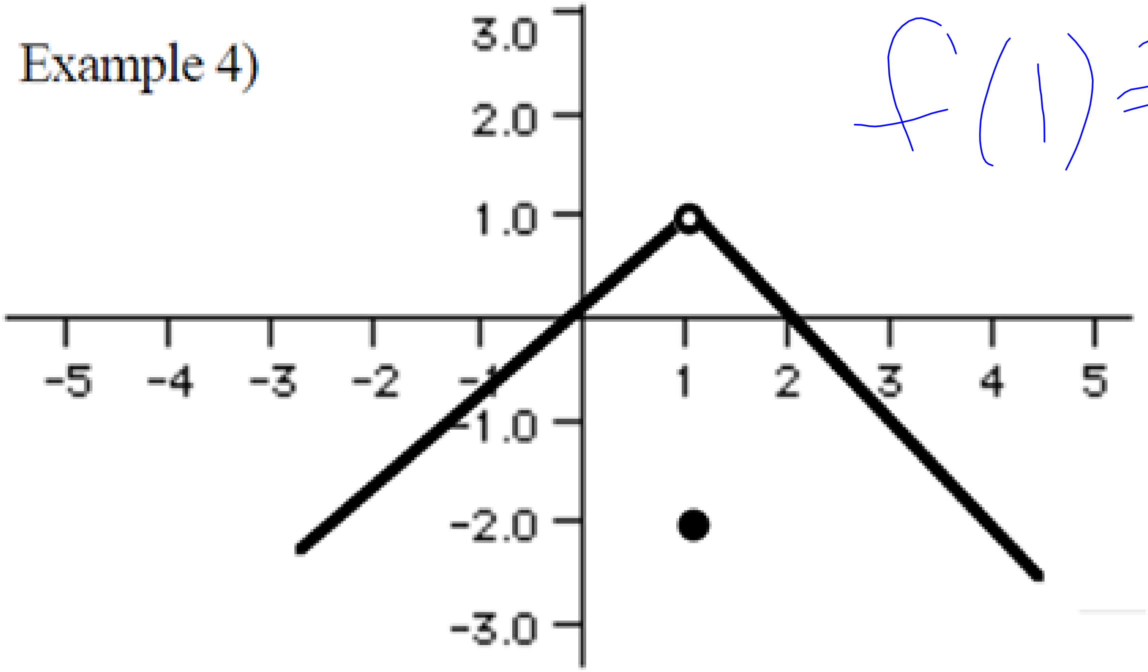
$$\lim_{x\to 1^+} f(x) = 1$$

$$\lim_{x\to 1} f(x) = 1$$

$$f(1) = \text{DNE}$$

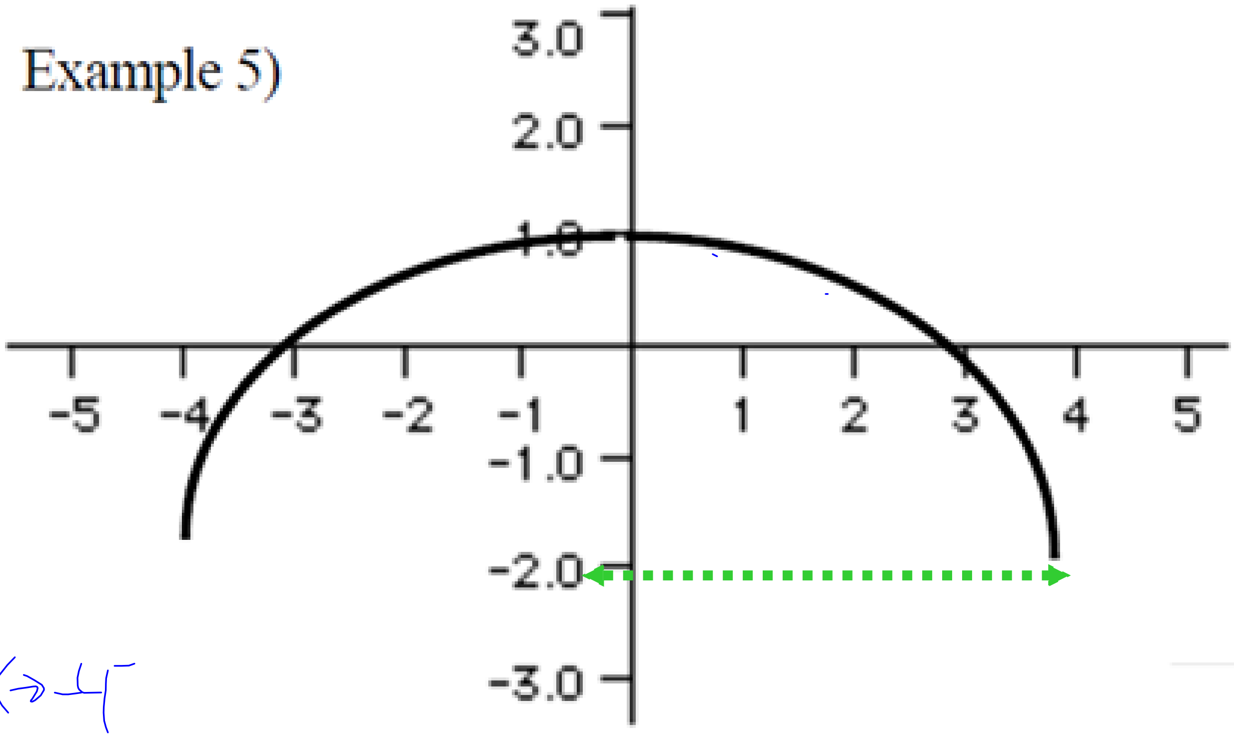
Example 4)

$f(1)=2$



Limits: Tabular and Graphical approach

Example 5)



$x \to 4^-$

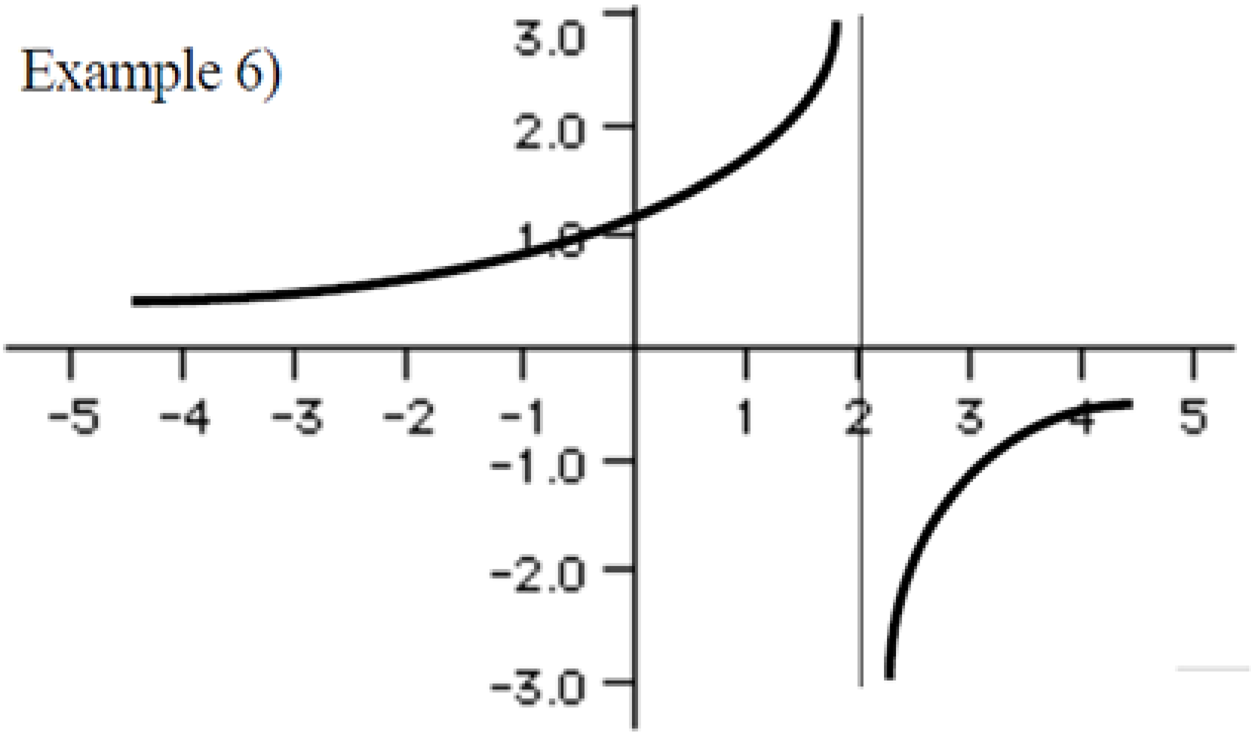
$\lim_{x \to 4^-} f(x) = -2$

$\lim_{x \to 4^+} f(x) = \text{DNE}$

$\lim_{x \to 4} f(x) = \text{DNE}$

$f(4) = -2$

Limits: Tabular and Graphical approach

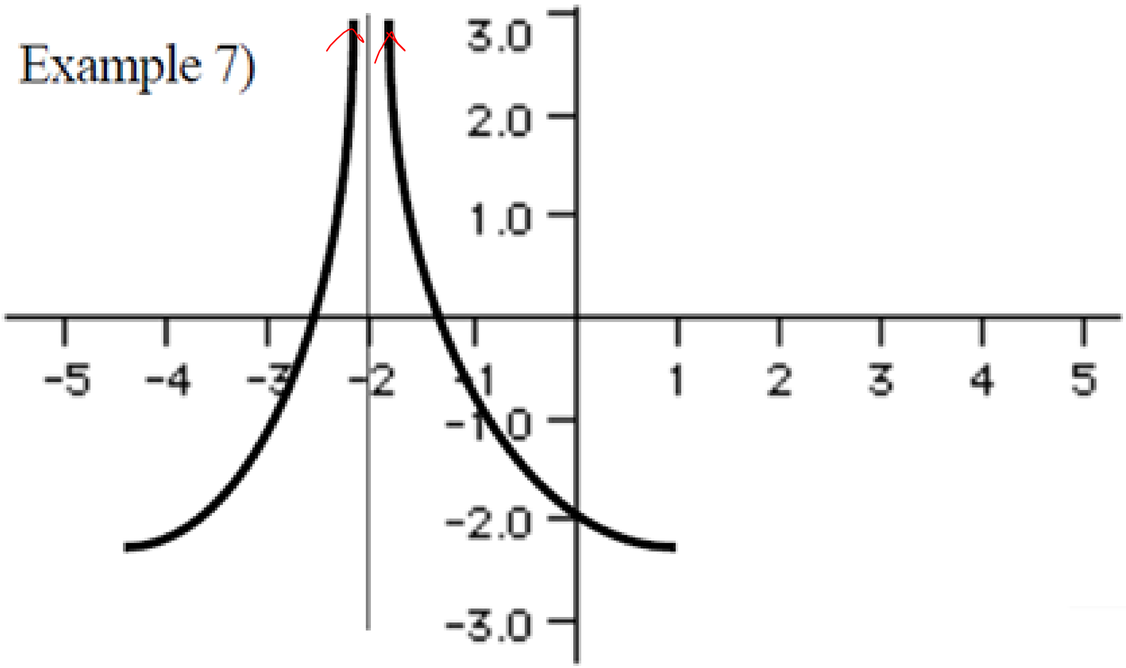


$$\lim_{x\to 2^-} f(x) = \infty \qquad \lim_{x\to 2^+} f(x) = -\infty$$

$$\lim_{x\to 2} f(x) = \text{DNE} \qquad f(2) = \text{Undef}$$

$$\lim_{x\to -\infty} f(x) = 0 \qquad \lim_{x\to \infty} f(x) = 0$$

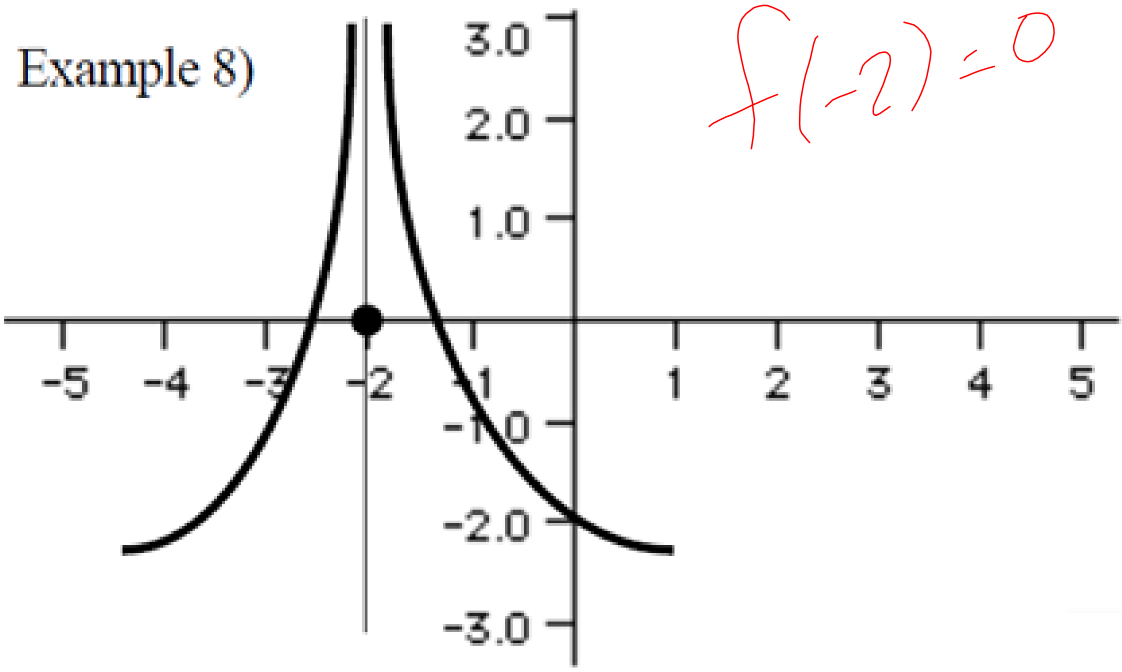
Example 7)



$\lim_{x\to -2^-} f(x) = \infty$ $\lim_{x\to -2^+} f(x) = \infty$

$\lim_{x\to -2} f(x) = \infty$ $f(-2) = $ undef.

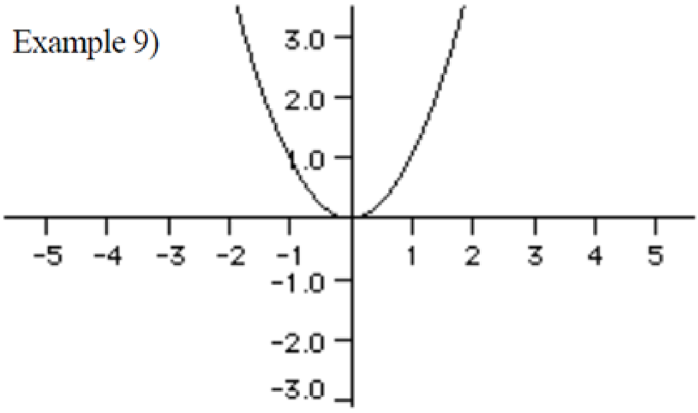
Example 8)



$f(-2)=0$

Limits: Tabular and Graphical approach

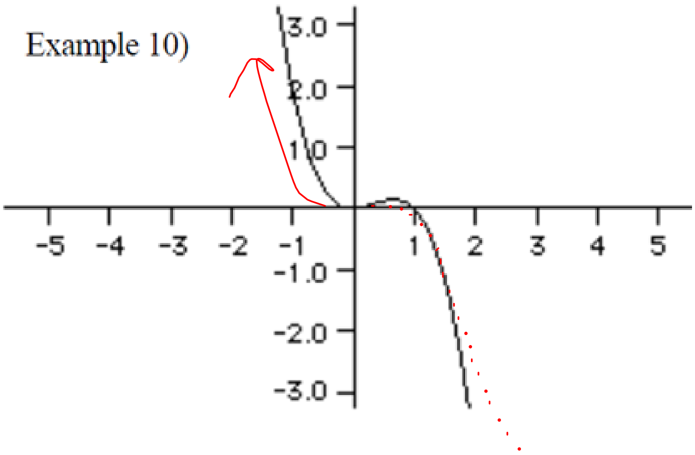
Example 9)



$$\lim_{x \to -\infty} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = \infty$$

Example 10)

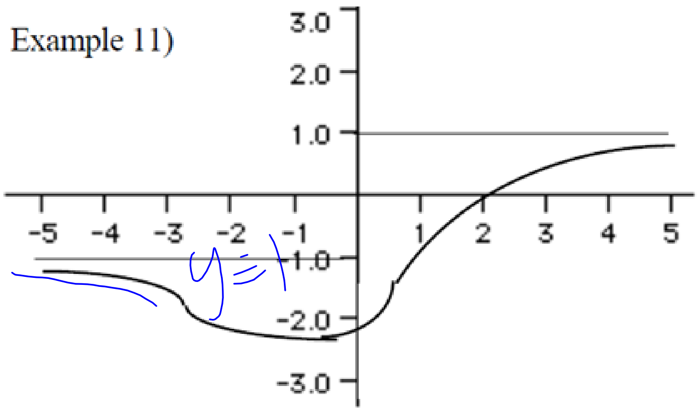


$$\lim_{x \to -\infty} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = -\infty$$

Limits: Tabular and Graphical approach

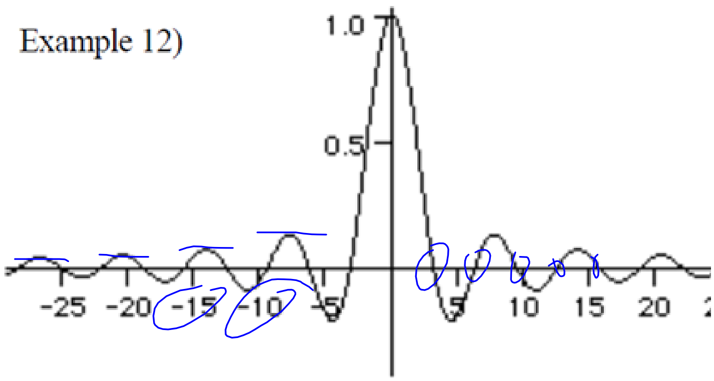
Example 11)



$$\lim_{x \to -\infty} f(x) = -1$$

$$\lim_{x \to \infty} f(x) = 1$$

Example 12)



$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to \infty} f(x) = 0$$

Tabular Approach

keeping in mind that BOTH SIDES MUST AGREE for a limit to exist, when we look at tables we need to look at both the left and right side.

Desmos

Example 13

$$\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3} \quad =-.25$$

x	-4	-3.1	-3.01	-3.001	-3	-2.999	$-.299$	-2.9	-2
$f(x)$	$-.2360$	$-.2484$	$-.2498$	$-.2499$	?	$-.25001$	$-.2501$	$-.2515$	$-.2679$

Example 14

$$\lim_{x \to 0} \cos\left(\frac{1}{x}\right) = \text{DNE}$$

x	$-.1$	$-.01$	$-.001$	$-.0001$	0	$.0001$	$.001$	$.01$	$.1$
$f(x)$	$-.8390$	$.8623$	$.5623$	$-.9521$	?	$-.9521$	$.5623$	$.8623$	$-.8390$

The function is oscillating close to zero so the limit does not exist

Example 15

$$\lim_{x\to 2}\frac{|x-2|}{x-2} = \text{DNE}$$

x	1	1.5	1.9	1.99	2	2.01	2.1	2.5	3
$f(x)$	-1	-1	-1	-1	?	1	1	1	1

$$\lim_{x\to 2^-}\frac{|x-2|}{x-2}=-1 \qquad \lim_{x\to 2^+}\frac{|x-2|}{x-2}=1$$

The limit does not exist because the left and right hand limit do not agree.