AP Calculus
Multiple Choice Monday #4

Name: Key
Date: ____________ Per: ____

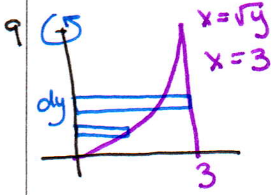
Problem	Section Name
1. The radius of a sphere is measured to be 5 cm with an error of ± 0.1 cm. Use differentials to approximate the error in volume. (A) $\pm\pi\ cm^3$ (B) $\pm 100\pi\ cm^3$ (C) $\pm 10\pi\ cm^3$ (D) $\pm 4\pi\ cm^3$ (E) $\pm 40\pi\ cm^3$ $V = \frac{4}{3}\pi r^3$ $dV = 4\pi r^2\, dr$ $= 4\pi (5)^2 (\pm .1)$ $= \pm 10\pi$	12
2. If $f(x) = \begin{cases} 2ax^2 + bx + 6, & x \le -1 \\ 3ax^3 - 2bx^2 + 4x, & x > -1 \end{cases}$ and is differentiable for all real values, then $b =$? (A) -13 (B) 0 (C) 45 (D) 55 (E) 110 cts derivative everywhere $f'(x) = \begin{cases} 4ax + b & x < -1 \\ 9ax^2 - 4bx + 4 & x > -1 \end{cases}$ $2a(-1)^2 + b(-1) + 6 = 3a(-1)^3 - 2b(-1)^2 + 4(-1)$ $2a - b + 6 = -3a - 2b - 4$ $b = -5a - 10$ $b = -5(-13) - 10$ $= 65 - 10$ $b = 55$ $4a(-1) + b = 9a(-1)^2 - 4b(-1) + 4$ $-4a + b = 9a + 4b + 4$ $13a + 3b = -4$ $13a + 3(-5a - 10) = -4$ $13a - 15a - 30 = -4$ $-2a = 26$ $a = -13$	6
3. An equation of the line normal to the graph of $y = \sqrt{3x^2 + 2x}$ at $(2, 4)$ is (A) $-4x + y = 20$ (B) $4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$ $y = (3x^2 + 2x)^{1/2}$ $y' = \frac{1}{2}(3x^2 + 2x)^{-1/2}(6x + 2)$ $y'(2) = \frac{6(2) + 2}{2\sqrt{3(2)^2 + 2(2)}} = \frac{14}{8} = \frac{7}{4}$ normal slope $= -4/7$ $y = -\frac{4}{7}(x - 2) + 4$ $7y = -4x + 8 + 28$ $\boxed{4x + 7y = 36}$	1, 7
4. If $\cos^2 x + \sin^2 y = y$, then $\frac{dy}{dx} =$ (A) $\frac{2\cos x \sin x}{2\cos y \sin y + 1}$ (B) $\frac{\cos x \sin x}{\cos y \sin y}$ (C) $\frac{2\cos x \sin x}{2\cos y \sin y - 1}$ (D) $\frac{\sin y \cos y}{1 - \cos x \sin x}$ (E) $\frac{2\cos y \sin y}{2\cos x \sin x - 1}$ $-2\cos x \cdot \sin x + 2\sin y \cdot \cos y \cdot \frac{dy}{dx} = \frac{dy}{dx}$ $-2\cos x \sin x = \frac{dy}{dx}(1 - 2\sin y \cos y)$ $\frac{dy}{dx} = \frac{-2\cos x \sin x}{1 - 2\sin y \cos y} = \frac{+2\cos x \sin x}{+(2\sin y \cos y - 1)}$	9

$x = \sqrt{y}$

5. Find the volume of the region formed by the curve $y = x^2$, the $x - axis$ and the line $x = 3$ when revolved about the $y - axis$. 31

(A) $\frac{3}{2}\pi$ (B) $\frac{9}{2}\pi$ (C) $\frac{27}{2}\pi$ (D) $\frac{81}{2}\pi$ (E) $\frac{243}{2}\pi$

washer $\pi \int_0^9 \underbrace{(0-3)^2}_{(-3)^2} - \underbrace{(0-\sqrt{y})^2}_{(-\sqrt{y})^2} dy$



$= \pi \int_0^9 9 - y \, dy$

$= \pi \left(9y - \frac{y^2}{2}\right]_0^9 = \pi\left[\left(9 \cdot 9 - \frac{9^2}{2}\right) - 0\right] = \pi\left(81 - \frac{81}{2}\right)$

$= \frac{81\pi}{2}$

*6. An open top cylinder has a volume of $125\pi \; in^3$. Find the radius required to minimize the amount of material to make the cylinder. 18

$125\pi = \pi r^2 h \rightarrow h = \frac{125\pi}{\pi r^2} = \frac{125}{r^2}$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Surface Area $= \pi r^2 + 2\pi r h$

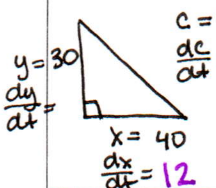
$S = \pi r^2 + 2\pi r \left(\frac{125}{r^2}\right)$

Calc $\underbrace{\frac{d}{dr}\left(\pi r^2 + \frac{250\pi}{r}\right)}$

Solve $\left(2\pi r - 250\pi r^{-2} = 0, r\right) \rightarrow r = 5$

*7. A 50 ft ladder is leaning against a building and being pulled to the ground, so the top is sliding down the building. If the rate the bottom ladder is being puller across the ground is 12 ft/sec, what is the rate the top of the ladder is sliding down the building when the top is 30 ft from the ground? 11

(A) 12 ft/sec (B) 9 ft/sec (C) 20 ft/sec (D) 9.6 ft/sec (E) 16 ft/sec



$c = 50$

$\frac{dc}{dt} = 0$

$x^2 + y^2 = c^2$

$x^2 + 30^2 = 50^2$

$x = 40$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$

$(40)(12) + (30)\frac{dy}{dt} = 50 \cdot 0$

$\frac{dy}{dt} = -16$

*8. If $f(x) = \left(1 + \frac{x}{20}\right)^5$ find $f''(40)$ 7

(A) 0.068 (B) 1.350 (C) 5.400 (D) 6.750 (E) 540.000

Calc $\frac{d^2}{dx^2}\left(\left(1 + \frac{x}{20}\right)^5\right) \Big| \, x = 40$

*9. A particle height at time $t \geq 0$ is given by $h(t) = 100t - 16t^2$. What is its maximum height? 19

(A) 312.500 (B) 156.250 (C) 78.125 (D) 6.250 (E) 3.125

↑y-value

$f1(x) := 100x - 16x^2$

Solve $\left(\frac{d}{dx}(f1(x)) = 0, x\right) \rightarrow x = 3.125$

$f1(3.125) = 156.25$