

## Problem

## Section Name

1. A side of a cube is measured to be 10 cm. Estimate the change in surface area of the cube when the side shrinks to 9.8 cm.

(A)  $+2.4 \text{ cm}^2$  (B)  $-2.4 \text{ cm}^2$  (C)  $-120 \text{ cm}^2$  (D)  $+24 \text{ cm}^2$  (E)  $-24 \text{ cm}^2$

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$$\text{Surface Area} = 6 \cdot \text{side}^2$$

$$A = 6s^2$$

$$dA = 12s \, ds$$

$$= 12(10)(-.2)$$

$$= -24$$

$$2. \frac{d}{dx} \left( \frac{x^3 - 4x^2 + 3x}{x^2 + 4x - 21} \right) = \frac{d}{dx} \left( \frac{(x^2 - x)(x - 3)}{(x+7)(x-3)} \right)$$

- (A)  $\frac{x^2 - x}{x+7}$  (B)  $\frac{x-1}{x-7}$  (C)  $\frac{x^2 - 14x + 7}{(x-7)^2}$  (D)  $\frac{2x^2 + 13x - 7}{(x+7)^2}$  (E)  $\frac{x^2 + 14x - 7}{(x+7)^2}$

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$$\begin{aligned} & \frac{x^2 - x}{2x - 1} \cancel{\times} \frac{x+7}{1} \rightarrow \frac{(2x-1)(x+7) - (x^2 - x)}{(x+7)^2} = \frac{2x^2 + 13x - 7 - x^2 + x}{(x+7)^2} \\ & = \frac{x^2 + 14x - 7}{(x+7)^2} \end{aligned}$$

$$3. \text{ Find } \lim_{x \rightarrow 0} \frac{2x^3 - 3 \sin x}{x^4}$$

- (A) -1 (B)  $\frac{1}{2}$  (C) 0 (D) 1 (E) Non existant

L'Hopital's Rule

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} \\ & = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} \\ & \text{for indeterminate limits} \end{aligned}$$

Since left + right do not agree

$$\begin{aligned} & \frac{2(0)^3 - 3 \sin(0)}{0^4} = \frac{0}{0} \\ & = \lim_{x \rightarrow 0} \frac{6x^2 - 3 \cos x}{4x^3} = \frac{(0(0)^2 - 3 \cos(0))}{4(0)^3} = \frac{-3}{0} \quad \begin{matrix} \text{Vert asymptote} \\ \checkmark \text{ must look at } 0^+, 0^- \end{matrix} \\ & \quad \frac{-3}{0^+} \rightarrow -\infty, \quad \frac{-3}{0^-} \rightarrow +\infty \end{aligned}$$

4. If  $f(x) = e^{3x}$ , then  $f''(\ln 3) =$   
 (A) 9 (B) 27 (C) 81 (D) 243 (E) 729

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$$f'(x) = e^{3x} \cdot 3$$

$$f''(x) = e^{3x} \cdot 3 \cdot 3$$

$$\begin{aligned} f''(\ln 3) &= 9e^{3 \ln 3} \\ &= 9e^{\ln 3^3} = 9e^{\ln 27} = 9 \cdot 27 = 243 \end{aligned}$$

5.  $\int_0^4 x^3 dx =$

- (A) 16 (B) 32 (C) 48 (D) 56 (E) 64

$$= \left[ \frac{x^4}{4} \right]_0^4 = \frac{4^4}{4} - \frac{0^4}{4} = 4^3 - 0 = 64 - 0$$

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\*6. If the position of a particle is given by  $x(t) = 2t^3 - 5t^2 + 4t + 6$ , where  $t > 0$ . What is the distance traveled by the particle from  $t = 0$  to  $t = 3$ ?

- (A)  $\frac{1}{27}$  (B)  $\frac{28}{27}$  (C) 20 (D) 21 (E)  $\frac{569}{27}$

$$\int_0^3 |(6t^2 - 10t + 4)| dt = 21.07407$$

$$\int |v(t)| dt \\ v(t) = (6t^2 - 10t + 4)$$

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\*7. The average value of the function  $f(x) = \ln^2 x$  on the interval  $[2, 4]$  is

- (A) -1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636

$$\frac{1}{b-a} \int_a^b f(x) dx \rightarrow \frac{1}{4-2} \int_2^4 (\ln x)^2 dx$$

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\*8. If  $f(x)$  is continuous and differentiable and  $f(x) = \begin{cases} ax^4 + 5x, & x \leq 2 \\ bx^2 - 3x, & x > 2 \end{cases}$ , then  $b =$

- (A) 0.5 (B) 0 (C) 2 (D) 6 (E) There is no value of b

$$a(2)^4 + 5(2) = b(2)^2 - 3(2) \\ 16a + 10 = 4b - 6 \\ 4a + 5 = b - \frac{3}{2} \\ 32a + 5 = 4b - 3$$

$$f'(x) = \begin{cases} 4ax^3 + 5 & x < 2 \\ 2bx - 3 & x > 2 \end{cases}$$

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$\begin{matrix} 3 \\ 7 \end{matrix}$

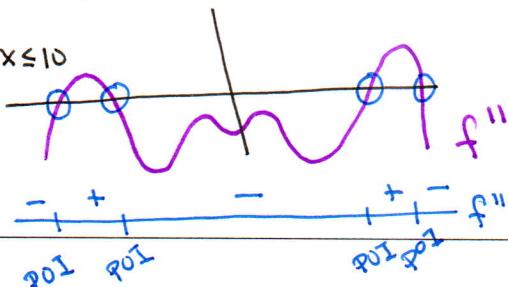
$$\text{Solve}\left(\left\{\begin{array}{l} 16a + 10 = 4b - 6 \\ 32a + 5 = 4b - 3 \end{array}\right., \{a, b\}\right)$$

\*9. The second derivative of a function  $f$  is given by  $f''(x) = x \sin x - 2$ . How many points of inflection does  $f$  have on the interval  $(-10, 10)$ ?

- (A) zero (B) two (C) four (D) six (E) eight

$$f''(x) := x \sin x - 2 \quad | -10 \leq x \leq 10$$

→ graph



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