

RWW #2

Key

Volume 1-26

a)

$$V = B \cdot h = (1.5)(1.5) \cdot h$$

Prism ← area of base

$$\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4}$$

$$\frac{dV}{dt} = \frac{3}{4} \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = -\frac{1}{4\sqrt{h}}$$

$$\frac{4}{3} \cdot \frac{-1}{4\sqrt{h}} = \frac{3}{4} \cdot \frac{dh}{dt} \cdot \frac{4}{3}$$

$$\frac{dh}{dt} = -\frac{1}{3\sqrt{h}}$$

b) $\frac{dh}{dt} = ?$ when $h = \frac{1}{2}$ full, full = 4
 $h = 2$ ←

$$\frac{dh}{dt} = \frac{-1}{3\sqrt{2}} = \boxed{\frac{-1}{3\sqrt{2}} \text{ in/mm}}$$

$$c) \quad \frac{dt}{dt} \frac{dh}{dt} = -\frac{1}{3\sqrt{h}} \cdot dt$$

$$\sqrt{h} \, dh = -\frac{1}{3\sqrt{h}} \, dt \cdot \sqrt{h}$$

$$\int h^{1/2} \, dh = \int -\frac{1}{3} \, dt$$

$$\frac{h^{3/2}}{3/2} = -\frac{1}{3} t + c$$

$$\frac{2}{3} h^{3/2} = -\frac{1}{3} t + c$$

$$\frac{2(4)^{3/2}}{3} = -\frac{1}{3} \cdot (0) + c$$

$$\frac{16}{3} = c$$

$$\frac{2}{3} h^{3/2} = -\frac{1}{3} t + \frac{16}{3}$$

$$h^{3/2} = -\frac{t}{2} + 8$$

$$h(t) = h = \left(-\frac{t}{2} + 8\right)^{2/3}$$

Cartridge full $\Rightarrow h(0) = 4$
 time \uparrow height of full ink

Separate

Integrate

Constant of integration

Initial Condition

$$4^{3/2} = \sqrt{4}^3 = 2^3 = 8$$

d) At what value of t does $h=0$?

$$h(t) = 0 = \left(-\frac{t}{2} + 8\right)^{2/3}$$

$$0 = -\frac{t}{2} + 8$$

$$\frac{t}{2} = 8$$

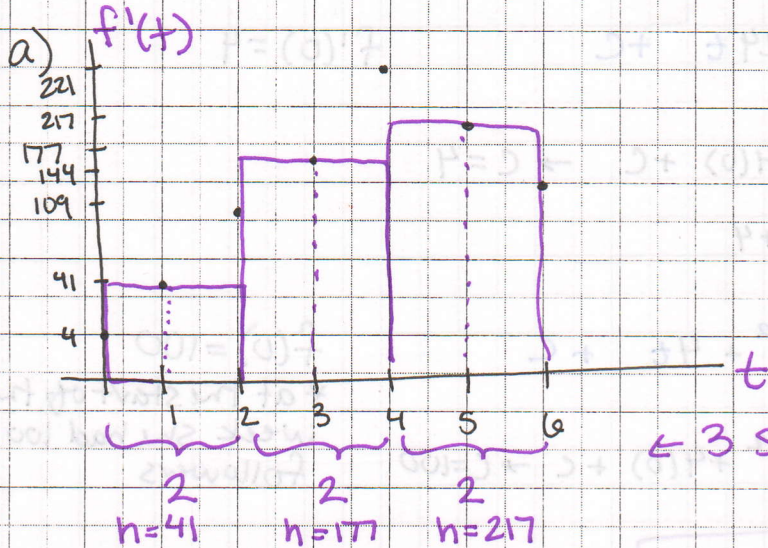
$$t = 16 \text{ hours}$$

after 16 hours, the Cartridge will be empty

RWW #2

Key

Volume 1-21



→ $f(t)$ = followers

$f'(t)$ = followers/day → growth rate

$f''(t)$ = followers/day² → rate of change of growth rate

want to accumulate followers

$$\int f'(t) dt = f(t)$$

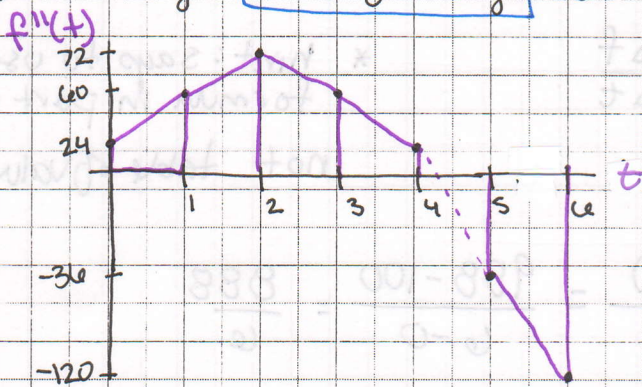
$$100 + \int_0^6 f'(t) dt = 2(41) + 2(177) + 2(217) = 870 + 100 = 970 \text{ followers at the end of the first week}$$

82 + 354 + 434

b) Average rate of change for growth rate

$$= \frac{1}{b-a} \int_b^a f''(t) dt$$

trapezoidal sum



$$\text{trap sum} = \frac{(24+60) \cdot 1}{2} + \frac{(60+72) \cdot 1}{2} + \frac{(72+60) \cdot 1}{2} + \frac{(60+24) \cdot 1}{2} + \frac{(24+(-36)) \cdot 1}{2} + \frac{(-36+(-120)) \cdot 1}{2}$$

$$42 + 66 + 66 + 42 + -6 + -78 = 132$$

$$\text{ave rate of change for } f'' = \frac{1}{6-0} \int_0^6 f''(t) dt = \frac{1}{6} (132) = 22 \text{ followers/day}^2$$

$$c) \int f''(t) = \int -12t^2 + 48t + 24$$

$$f'(t) = -\frac{12t^3}{3} + \frac{48t^2}{2} + 24t + C$$

$$f'(0) = 4$$

$$4 = -4(0)^3 + 24(0)^2 + 24(0) + C \rightarrow C = 4$$

$$\int f'(t) = \int -4t^3 + 24t^2 + 24t + 4$$

$$f(t) = -\frac{4t^4}{4} + \frac{24t^3}{3} + \frac{24t^2}{2} + 4t + C$$

$$f(0) = 100$$

* at the start of the week she had 100 followers

$$100 = -(0)^4 + 8(0)^3 + 12(0)^2 + 4(0) + C \rightarrow C = 100$$

$$f(t) = -t^4 + 8t^3 + 12t^2 + 4t + 100$$

$$\text{Day } 6 \Rightarrow t = 6$$

$$f(6) = -6^4 + 8 \cdot 6^3 + 12(6)^2 + 4(6) + 100$$

$$f(6) = 988 \text{ followers}$$

$$d) \text{ Average growth rate} = \frac{\Delta f}{\Delta t}$$

↑
2 points to get f'

* hint: says to use formula in part c
not table of values

$$\text{Average growth} = \frac{f(6) - f(0)}{6 - 0} = \frac{988 - 100}{6 - 0} = \frac{888}{6}$$

$$= 148 \text{ followers/day} > 125$$

Twitter will flag Aunt Twidiot.