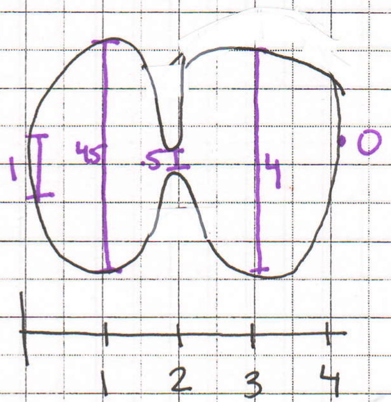
RWW #3 Key

Volume 1-25

a)



$$\text{Area} = \frac{(1+4.5)}{2}\cdot 1 + \frac{(4.5+.5)}{2}\cdot 1 + \frac{(.5+4)}{2}\cdot 1 + \frac{(4+0)}{2}\cdot 1$$

$$= \frac{1}{2}\left(1 + 2(4.5) + 2(.5) + 2(4) + 0\right)$$

$$= \frac{1}{2}\left(1 + 9 + 1 + 8\right) = \frac{1}{2}\cdot 19 = \frac{19}{2}\ \text{in}^2$$

$$\boxed{\text{Area} \approx 9.5\ \text{in}^2}$$

b) $\text{Vol. flow} = A \cdot v(t) = 9.5 \cdot v(t)$

Ave value of Volumetric flow $= \dfrac{1}{b-a}\displaystyle\int_b^a \text{Vol. flow}$

Calc $\dfrac{1}{60-0}\displaystyle\int_0^{60} 9.5\left(15 + 3\sin(.2t - 30)\right)dt = 141.29809$

Average value of Volumetric flow $= 141.298\ \text{in}^3/\text{second}$

c) $\text{Area} = \int_0^4 \text{height} \cdot dt$

$$= \int_0^4 -x^4 + 8x^3 - 20x^2 + 16x + 1 \; dx$$

$$= \frac{-x^5}{5} + \frac{8x^4}{4} - \frac{20x^3}{3} + \frac{16x^2}{2} + 1x \Big]_0^4$$

$$= -\frac{1}{5}x^5 + 2x^4 - \frac{20}{3}x^3 + 8x^2 + x \Big]_0^4$$

$$= -\frac{1}{5}(4)^5 + 2(4)^4 - \frac{20}{3}(4)^3 + 8(4)^2 + 4 - (0)$$

$$= \frac{-1024 \cdot 3}{5 \cdot 3} + 512 - \frac{1280 \cdot 5}{3 \cdot 5} + 128 + 4 = 644 + \frac{-9472}{15}$$

$$\frac{-3072}{15} \qquad \frac{-6400}{15} \qquad = \frac{9660}{15} - \frac{9472}{15} = \boxed{\frac{188}{15} \text{ in}^2}$$

d) $\text{Vol flow} = \frac{188}{15} \cdot v(t) \;\; \text{in}^3/\text{min}$

times $\rightarrow$ $\int_a^b \text{Vol flow} = \text{volume in}^3$

$$\boxed{\int_0^K \frac{188}{15} \cdot v(t)\, dt = 6 \cdot 12^3}$$

6 cubic ft

→ need to convert to inches

$$6 \text{ ft}^3 \cdot \frac{(12 \text{ in})^3}{1 \text{ ft}^3} = 6 \cdot 12^3$$

RWW #3 Key

Volume 1-18

a) max distance = max y ($y'=0$ (+) to (−)) from $t=0$ to $t=\pi$

$y=-300e^{-t}(\cos t-1)$

$-300e^{-t}$ ↗ $\cos t - 1$

$300e^{-t}$ $-\sin t$

$y' = 300e^{-t}(\cos t+1) + 300e^{-t}\cdot\sin t$

$= \underbrace{300e^{-t}}_{\neq 0}\underbrace{(\cos t - 1 + \sin t)}_{=0}$

$\sin t + \cos t = 1$ ← Calc will solve or graph + find intersection

$t=\pi/2$

$f1(x) = \sin x + \cos x$ $f2(x) = 1$

menu → analyze → Intersection

Check boundaries

t	0	$\pi/2$	π
y(t)	0	$\frac{300}{e^{\pi/2}}$ ≈62.36389	$\frac{600}{e^{\pi}}$ ≈ 25.9284

max

OR

y, y': (0 + $\pi/2$ − π)

max

$\boxed{\text{max distance is } 62.363 \text{ ft}}$ at $t=\pi/2$

b) How far → accumulate distance → $\int |v(t)|\,dt$

Calc $\int_0^{\pi} \left|300e^{-t}(\cos t - 1 + \sin t)\right| dt \approx 98.79948$ ft

She travels a total of 98.799 ft.

c) $y' = v(t) = 300e^{-t}(\sin t + \cos t - 1)$

$$v\left(\tfrac{3\pi}{4}\right) = \underbrace{300e^{-\frac{3\pi}{4}}}_{>0}\underbrace{\left(\underbrace{\sin\left(\tfrac{3\pi}{4}\right)}_{>0} + \underbrace{\cos\left(\tfrac{3\pi}{4}\right)}_{<0}\right.}_{-1}\left.-1\right) < 0$$

= but opposite sign

or, if you did a sign chart for part a

$300e^{-t}$ $\sin t + \cos t - 1$

$-300e^{-t}$ $\cos t - \sin t$

$$y''(t) = a(t) = -300e^{-t}(\sin t + \cos t - 1) + 300e^{-t}(\cos t - \sin t)$$

$$= -300e^{-t}\left[\sin t + \cos t - 1 - (\cos t - \sin t)\right]$$

$$= -300e^{-t}(2\sin t - 1)$$

$$a\left(\tfrac{3\pi}{4}\right) = \underbrace{-300e^{-3\pi/4}}_{<0}\left(\underbrace{2\sin\left(\tfrac{3\pi}{4}\right) - 1}_{>0}\right) < 0$$

Since both $v\left(\frac{3\pi}{4}\right), a\left(\frac{3\pi}{4}\right)$ are negative, they are working together and chessie is speeding up.

d) max acceleration $\rightarrow a'(t) = 0$

$a(t) = -300e^{-t}(2\sin t - 1)$

$-300e^{-t}$ $2\sin t - 1$

$300e^{-t}$ $2\cos t$

$a'(t) = 300e^{-t}(2\sin t - 1) - 300e^{-t}(2\cos t)$

$$\boxed{0 = 300e^{-t}(2\sin t - 1 - 2\cos t)}$$