Secant and Tangent Lines

Write down what you know (or think) about each:

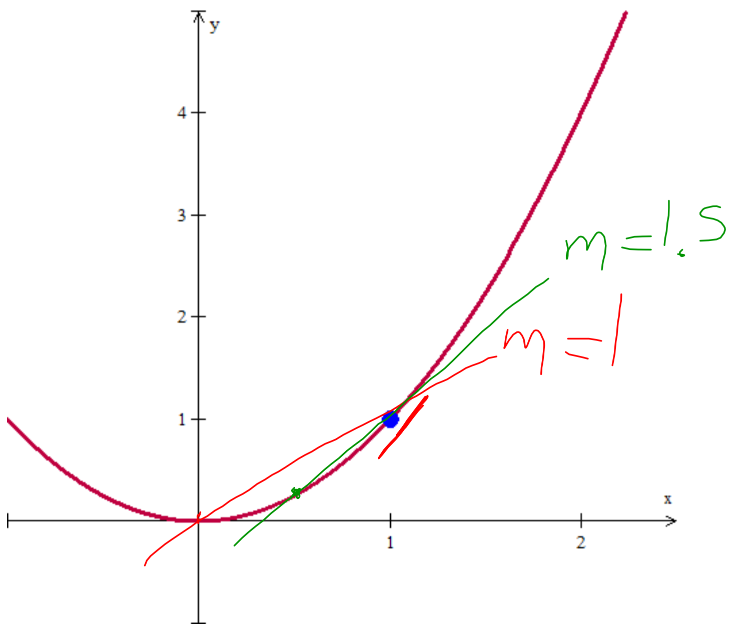
Tangent

Secant

Slope

Example 1: Lets try to find the slope of the line tangent to $f(x)=x^2$ at the point $x=1$

x	$m_{\sec}$
0	1
.5	1.5
.8	1.8
.97	1.97
1	?
1.05	2.05
1.5	2.5
2	3



Why can't we find the slope of the line between x=1 and x=1?

because slope requires the use of TWO points

What is your best guess at the slope of the tangent line at x=1?

it appears that both sides are approaching 2

Example 2: Suppose you leave your house at 10 am for a trip to Disneyland. You arrive at Disneyland at 12 pm. The trip is a total of 100 miles.

How fast are you going at 11 am?

some students said 50 mph

I am going to travel around the room is a pretend trip to "disneyland". back half of the room close your eyes, front half look at how fast(or slow) is am going when i get to halfway.

Now switch, front close eyes, back look at how fast i am going when i get to half way.

Would describe my pace as fast or slow?

Why is there a problem?

Can you answer the question how fast was i going at 11 am?

no, we do not have enough information

Instantaneous Velocity - slope of the TANGENT

Average Velocity - slope of the SECANT

What is the average velocity between 10 am and 12 pm?

50 mph

Find each of the average velocities (miles per minute) given the distance traveled between 11 am and the given time.

Time	Distance traveled since 11	Average velocity from 11 to t	
11:30	24 miles	.8 mpm	48 mph
11:15	13 miles	.86	52 mph
11:10	10 miles	1	60
11:05	5.5	1.1	66
11:02	2	1	60
11:01	.8	.8	48
11:00:30	.53	1.06	63
11:00			

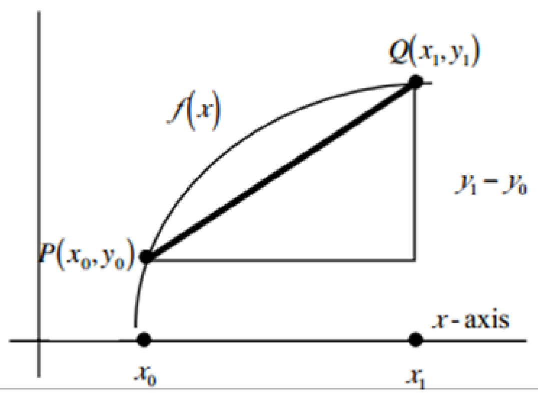
Can you find the INSTANTANEOUS velocity at 11? no, but we can estimate

As we get closer time/distance measurements, can we estimate the velocity at 11? yes

What would be a good estimate? 63 mph

These are all still what type of lines? secant

Secant and Tangent Lines



Let us now take the two points and give them general coordinates $P(x_0, y_0)$ and $Q(x_1, y_1)$

We draw the right triangle below the curve. Note that the length of the horizontal line is $x_1 - x_0$ and the length of the vertical line is $y_1 - y_0$

We now find the slope of the secant line PQ and denote it as m_{sec}.

$$m_{sec} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0}$$

note: $y = f(x)$

we can rewrite slope as

$$m_{sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

let $h = x_1 - x_0$ $\qquad \Delta x = x_1 - x_0$

we can rewrite this as

$$x_1 = x_0 + h$$

which lets us write $$m_{sec} = \frac{f(x_0 + h) - f(x_0)}{\cancel{x_0 + h - x_0}}$$

h

What was a key idea in calculus that separates it from prior math classes?

limits!

Can h=0? no

Why not?

because then we would be using one point for slope instead of two.

Formulas you need to know:

Slope of the secant line

$$m_{sec} = \frac{f\left(x_1\right) - f(x_0)}{x_1 - x_0}$$

Slope of the tangent line

$$m_{tan} = \frac{f\left(x_0 + h\right) - f(x_0)}{h}$$ as h gets infinitely close to 0

Point-Slope equation of a line

$$y = m\left(x - x_1\right) + y_1$$

Example 3: for the function $y = 5x - 2$ find the following:

a) the slope of the secant line between x=-1 and x=4

$f(4) = 5(4) - 2$
$= 18$
$f(-1) = 5(-1) - 2$
$= -7$

$m_{sec} = \frac{f(4) - f(-1)}{4 - -1}$
$= \frac{18 + 7}{4 + 1}$
$= \frac{25}{5} = 5$

b) the slope of the tangent line at x=2

$f(2+h) = 5(2+h) - 2$
$= 5h + 8$

$f(2) = 5(2) - 2$
$= 8$

$$m_{tan} = \frac{f(2+h) - f(2)}{h}$$

$$m_{tan} = \frac{5h + 8 - 8}{h}$$

$$= \frac{5h}{h} = 5$$

~~$5 \cdot 0 = 5$, $\frac{0}{0} \stackrel{?}{=} 5$~~

c) the equation of the tangent line at x=2

$(2, 8)$ $m = 5$

$y = 5(x - 2) + 8$
$= 5x - 2$

Example 4: for the function $y = x^2 + 1$ find the following:

a) the slope of the secant line between x=1 and x=3

$$m_{sec} = \frac{10-2}{3-1} = \frac{8}{2} = 4$$

b) the slope of the tangent line at x=2

$$f(2+h) = (2+h)^2 + 1 \qquad f(2) = 5$$

$$= (2+h)(2+h) + 1$$

$$= h^2 + 4h + 5$$

$$m_{tan} = \frac{h^2 + 4h + 5 - 5}{h}$$

$$= \frac{\cancel{h}(h+4)}{\cancel{h}} = h + 4 \rightarrow 4$$

$$h \to 0$$

c) the equation of the tangent line at x=2

$$y = 4(x-2) + 5$$

$$= 4x - 3$$

Example 5: for the function $y = x^2 + 4x - 1$ find the following:

a) the slope of the secant line between x=1 and x=3

$$m_{sec} = \frac{20-4}{3-1} = 8$$

b) the slope of the tangent line at x=2

$$f(2+h) = (2+h)^2 + 4(2+h) - 1$$
$$= h^2 + 8h + 11$$

$$f(2) = 11$$

$$m_{tan} = \frac{f(2+h) - f(2)}{h}$$

$$= \frac{h^2+8h}{h} = \frac{\cancel{h}(h+8)}{\cancel{h}}$$

$$= h + 8 \rightarrow 8$$

c) the equation of the tangent line at x=2

$$y = 8(x-2) + 11$$

$$y = 8x - 5$$

example 5 1/2) Find the slope of the tangent for the equation $y=\frac{1}{x}$

b) tan Slope @ $x=2$

$$\frac{\frac{2\cdot 1}{2(2+h)} - \frac{1(2+h)}{(2)(2+h)}}{h}$$

$$\frac{\left(\frac{(2)-(2+h)}{4+2h}\right)}{h} = \frac{1}{\cancel{h}}\left(\frac{-1\,\cancel{h}}{4+2h}\right)$$

$$= \frac{-1}{4+2h} \Rightarrow \frac{-1}{4}$$

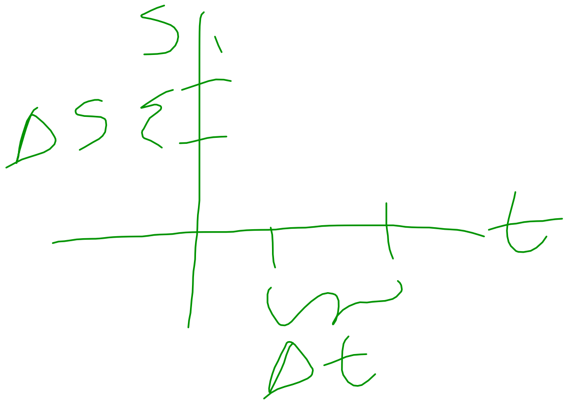
Secant and Tangent Lines

We can do the same thing with position functions, usually labeled s(t) or x(t).

position function gives the location of a particle in terms of a single dimension (usually in the x direction)

Given a value of time, t, s(t) gives a particles position.

$$\frac{\Delta s}{\Delta t}$$



If we use two values of t, which type of velocity are we measuring?

ave, sec

What about 1 value of t?

ins, tan

Recall: what is the difference between speed and velocity?

velocity has direction

Example 6: If $s(t) = 3t + 1$ is a measure of feet traveled per second, find

a) the average velocity between t=0 and t=3

$$V_{ave} = \frac{10-1}{3-0} = 3 \text{ ft/sec}$$

b) the instantaneous velocity at t=2

$$V_{ins} = \frac{s(2+h) - s(2)}{h}$$

$$= \frac{3h + 7 - 7}{h}$$

$$= \frac{3\cancel{h}}{\cancel{h}}$$

$$V_{ins} \Rightarrow 3 \text{ ft/sec}$$

Example 7: If $s(t) = t^2 - 2t$ is a measure of feet traveled per second, find

$$S(t) = t^2 - 2t$$

a) the average velocity between t=0 and t=2

$$V_{ave} = \frac{0-0}{2-0} = \frac{0}{2} = 0 \text{ ft/sec}$$

b) the instantaneous velocity at t=2

$$S(2+h) = h^2 + 2h$$
$$S(2) = 0$$

$$V_{ins} = \frac{h^2+2h}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$V_{ins} \rightarrow 2 \text{ ft/sec}$$

Assignment #1 due:

Thurs 8/1

Reminder: Quiz Friday 8/2