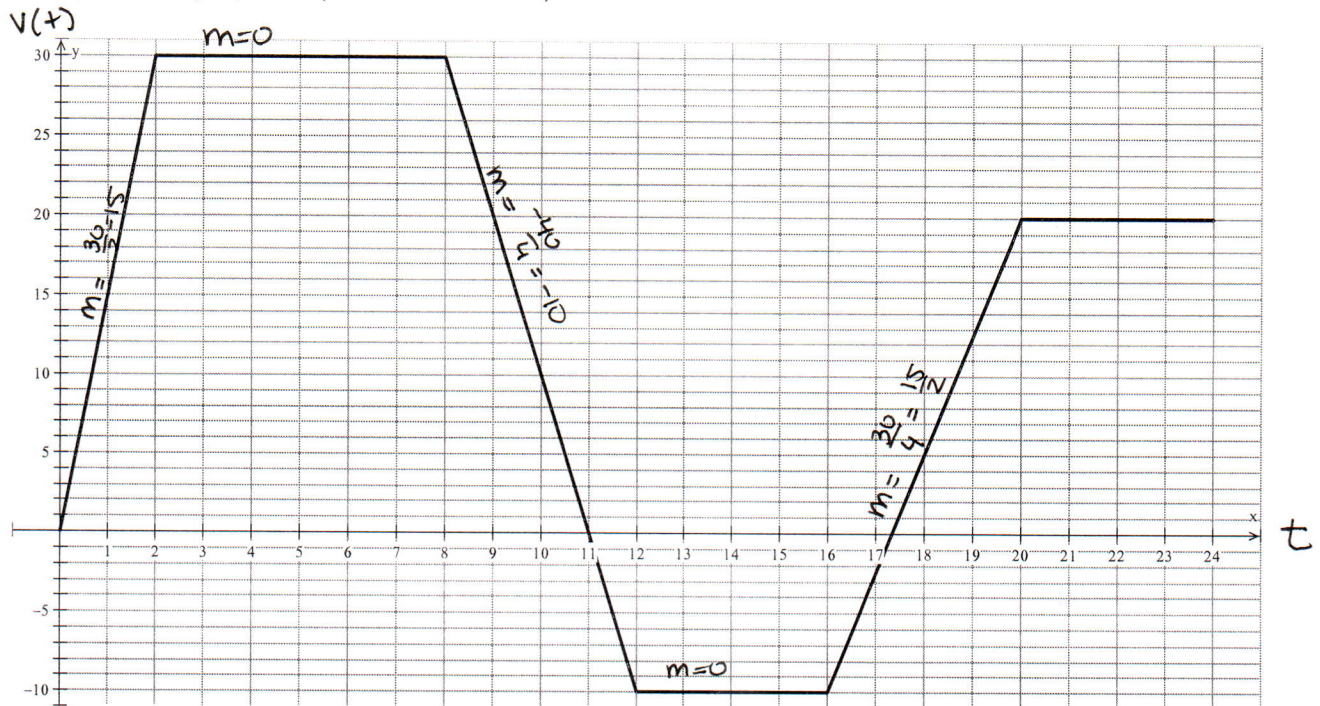The following is a graph of the a single Zombie's velocity during the first 24 hours of the Zombie Apocalypse caused by an AP Calculus student bubbling the "DO NOT BUBBLE" portion of the mock AP test. (I warned you, I told you not to bubble!)

(a) Write the piecewise function for $v(t)$

$$v(t) = \begin{cases} ① & 15t & 0 \le t < 2 \\ ② & 30 & 2 \le t < 8 \\ ③ & -10(t-8)+30 & 8 \le t < 12 \end{cases}$$

④ -10 $12 \le t < 16$

⑤ $\frac{15}{2}(t-16)-10$ $16 \le t < 20$

⑥ 20 $20 \le t \le 24$

(b) At what times during the interval $0 \le t \le 24$, if any, does the zombie change direction? Give a Calculus-based reason for your answer

$v(t) = 0$ at $t = 0, 11, 17\frac{1}{3}$

$\frac{15}{2}(t-16)-10 = 0$

$\frac{15}{2}t - 120 - 10 = 0$

at $t = 11, 17\frac{1}{3}$

$\frac{15}{2}t = 130 \cdot \frac{2}{15}$

$= 17\frac{1}{3}$

Since V changes Sign

Right +   Left −   Right +

$v(t)$ 0 11 $17\frac{1}{3}$

(c) During what time interval(s) is the acceleration equal to zero? Justify with Calculus.

$a(t) = v'(t) = 0$

↳ when is the slope of V = 0 ?

$2 < t < 8$ and $12 < t < 16$

(d) If $s(4) = 50$, then write the equation of the tangent to the graph of $s(t)$, the position function, at $t = 4$.

$\boxed{y = 30(x-4) + 50}$

$\hookrightarrow (4, 50) \quad s'(4) = V(4) = 30$
$\qquad$ Point $\qquad$ Slope $\qquad \hookleftarrow$ graph

(e) Use the tangent line to the graph of $s(t)$ at $t = 4$ to estimate the zombie's position at $t = 4.5$

$S(4.5) \approx y(4.5) = 30(4.5 - 4) + 50$
$\qquad\qquad = 30(\frac{1}{2}) + 50$
$\qquad\qquad = 15 + 50 = 65$

$\boxed{65m}$

(f) During what open intervals is the graph of $s(t)$, the position function concave down. Justify with Calculus.

$CD \to S'' < 0$

$S' = V$
$S'' = V'$ when are the slopes of $V < 0$

$\boxed{8 < t < 12}$

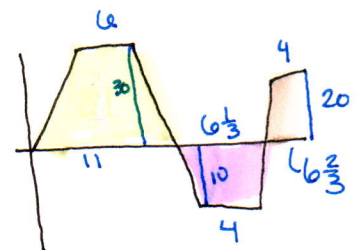(g) During what time(s) in the open interval $0 \le t \le 24$ is $a(t)$ undefined? [Justify]

$a(t) = V'(t)$ when is the slope of $V(t)$ undefined?

at the corners/cusp's $\boxed{t = 2, 8, 12, 16, 20}$

(h) How far from home was the zombie at $t = 24$?

$\int_0^{24} V(t)\, dt = 225 - \frac{155}{3} + \frac{320}{3} = \boxed{280 \text{ ft}}$

$(\frac{6 + 11}{2})30 = 225$

(i) What is the total distance the zombie traveled?

$\int_0^{24} |V(t)|\, dt = 225 + \frac{155}{3} + \frac{320}{3} = \frac{1150}{3} \text{ ft}$

$= \boxed{383.\overline{3} \text{ ft}}$

$(\frac{4 + \frac{19}{3}}{2})10 = \frac{155}{3}$

$(\frac{4 + \frac{20}{3}}{2})20 = \frac{320}{3}$